Supplementary Material for: Do We Exploit all Information for Counterfactual Analysis? Benefits of Factor Models and Idiosyncratic Correction

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Abstract

In this Supplementary Material we provide a number of additional results to the paper "Do We Exploit all Information for Counterfactual Analysis? Benefits of Factor Models and Idiosyncratic Correction". In addition to the proof of the theoretical results in the above mentioned paper, we also report additional empirical results.
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1 Introduction

This is a supplement to the paper "Do We Exploit all Information for Counterfactual Analysis? Benefits of Factor Models and Idiosyncratic Correction". The document is organized as follows. In Section 2 we describe the algorithm used to split the cities into the treatment and control groups. Section 3 contains additional empirical results. More specifically, in Section 3.2 we compare the empirical results when the ArCo methodology of Carvalho, Masini, and Medeiros (2018) and the Principal Component Regression (PCR) as in Gobillon and Magnac (2016) are used to estimate the counterfactuals. In Section 3.3 we evaluate different approaches to model trending behavior in the data, while in Section 3.4 we present the results at a state-level aggregation. The proof of the main result in the paper is presented in Section 4.

2 Randomization Algorithm

In this section we describe the algorithm used to split the municipalities into two different groups according to a set of characteristics. Once the groups are formed we randomly label them as treatment and control groups.

Let $\boldsymbol{Z}$ be a $n \times J$ matrix of municipalities' variables, where each column j is a different characteristic (covariate) and each row i is a municipality, n is the number of municipalities and J is the number of covariates. We consider the following variables: human development index, employment, GDP per capita, population, female population, literate population, average household income (total), household income (urban areas), number of stores, and number of convenience stores.

The goal is to match the average of each characteristic of the treatment group with the control group. Once each group of municipalities is created, each group is further divided into two other groups, resulting in four different sets of municipalities. The experiments were carried on different combinations of the groups. In the paper, we report only one set of the experiments.

The optimization problem is defined as

$$\widehat{\boldsymbol{\alpha}} = \underset{\boldsymbol{\alpha}}{\arg\min} \frac{1}{J}\sum_{j=1}^{J}\left|\frac{1}{\sum_{i=1}^{n}\alpha_i}\sum_{i=1}^{n}\alpha_i Z_{i,j} - \frac{1}{\sum_{i=1}^{n}(1-\alpha_i)}\sum_{i=1}^{n}(1-\alpha_i)Z_{i,j}\right|$$

$$\text{subject to: } \sum_{i=1}^{n}\alpha_i = K \text{ and } \alpha_i \in \{0,1\}\,\forall\, i,$$

where $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)'$, $\alpha_i = 1$ if municipality i belongs to the first group and $\alpha_i = 0$ otherwise; K is the number of municipalities in the first group. The optimization problem above can be transformed into a mixed integer program.

3 Additional Empirical Results

In this section we report a number of additional empirical results with the aim of showing the robustness and advantages of the proposed methodology.

3.1 Additional Plots

Figures S.1–S.4 display relevant data for Products II-V. Panel (a) in the figures reports the daily sales at each group of municipalities (all, treatment, and control) divided by the number of stores in each group. More specifically, the plot shows the daily evolution of $q^{(j)}_{\text{all},t\cdot} = \frac{1}{s}\sum_{i=1}^{n} q^{(j)}_{it}$, $q^{(j)}_{\text{control},t\cdot} = \frac{1}{s_0}\sum_{i=1}^{n_0} q^{(j)}_{it}$, and $q^{(j)}_{\text{treatment},t\cdot} = \frac{1}{s_1}\sum_{i=n_0+1}^{n} q^{(j)}_{it}$. The plot shows the data before and after price changes and the intervention date is represented by the horizontal line. Panels (b) and (c) display the distribution across municipalities of the time averages of $\widetilde{q}^{(j)}_{it}$, before and after the intervention and for the treatment and control groups, respectively. Panels (d) and (e) present fan plots for the evolution of $\widetilde{q}^{(j)}_{it}$. The black curves there represent the cross-sectional means over time.

Figures S.5–S.8 display some estimation results. For each product, Panel (a) in the figures displays a fan plot of the p-values of the ressampling test for the null hypothesis $\mathcal{H}_0 : \delta_t = 0$ for each given t after the treatment, using the test statistic $\phi(\widehat{\delta}_t) = |\widehat{\delta}_t|$, which is the same as using the test statistic $\widehat{\delta}^2_t$. The black curve represents the cross-sectional median across time t. Panel (b) shows an example for one municipality. The panel shows the actual and counterfactual sales per store for the post-treatment period. 95% confidence intervals for the counterfactual path are also displayed.

Figure S.9 displays the distribution of the daily evolution of the inventory of each product across different municipalities.

3.2 Effects of Additional Information: `ArCo`, `PCR`, and `FarmTreat`

We report the estimation results when either the `ArCo` methodology of Carvalho, Masini, and Medeiros (2018) or principal component regression (PCR) in the spirit of Gobillon and Magnac (2016) are used. For the ArCo methodology we construct counterfactuals by estimating a LASSO regression of $\widetilde{q}_{it}^{(j)}$ on the values of $\widetilde{q}_{kt}^{(j)}$, where $k \in \{1, \ldots, n\}/i$. Note that we do not include any other regressor. For PCR, we consider the first two stages of the `FarmTreat` methodology.

The `ArCo` results are reported in Tables S.1 and S.2 while the results for the PCR method are shown in Tables S.3 and S.4. Some interesting facts emerge from the tables. First, the `ArCo` and `FarmTreat` show similar results, with the later having a slightly better pre-intervention fit. One key difference, however, is the substantially smaller number of municipalities with significant intervention effects when the ArCo methodology is considered. Comparing the PCR approach and the `FarmTreat`, we can clearly see an improvement in the pre-intervention fit, as expected. As in the `ArCo` method, the `PCR` approach yields a smaller fraction of cities with significant effects of the price changes. Finally, one important point to highlight is that all three methods suggests that on average the current prices must be decreased.

3.3 Effects of Trends

Tables S.5 and S.6 report the results of the `FarmTreat` methodology is used without detrending the data in the first step. Compared to the baseline results presented in Tables 6 and 7 in the main text we highlight the following facts. First, the counterfactual model adjustment is similar with only marginal differences concerning the pre-intervention R-squared. Second, without detrending, the average treatment effects are smaller but the rejection rates are higher. Third, the number of municipalities where the estimated Δ has the correct sign and is statistically significant at the 10% level is much smaller when we do not include a linear trend in the first step of the methodology, specially in the case of Product V. We note that for this last product, the recommendation is a price increase and not decrease. For the other four products, the conclusions are similar as the baseline case.

3.4 State-Level Aggregation

Tables S.7 and S.8 report the results of the `FarmTreat` methodology applied to data aggregated at the state level. The control and treatment groups at the state-level are constructed by aggregating the untreated and treated municipalities in each state, respectively. From the tables we see that for Products IV and V we do not find significant effects at the state level. This is mainly due to heterogeneity across municipalities within each state. On the other hand, for Products I, II and III we find significant effects of price changes on sales. On the average, the optimal price for Products III and V are higher than the actual ones, whereas for Product IV the `FarmTreat` method indicates that on average the prices should be reduced. However, even for this products the effects are significant in only a fraction of states. These results, corroborates the huge municipality heterogeneity.

3.5 Before-and-After Estimation

Table S.9 reports estimation the average treatment effect using the before-and-after estimator. In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a variety of different statistics. We consider the distribution over the treated municipalities.

4 Proof of the Main Result

Before proving our main result, we define below the compatibility constant for convenience.

Definition 1. *For a $(n \times n)$ matrix $\boldsymbol{M}$, a set $\mathcal{S} \subseteq [n]$ and a scalar $\zeta \geqslant 0$, the compatibility constant is given by*

$$\kappa(\boldsymbol{M}, \mathcal{S}, \zeta) := \operatorname{inf}\left\{\frac{\|\boldsymbol{x}^T\boldsymbol{M}\boldsymbol{x}\|}{\sqrt{|\mathcal{S}|}}\|\boldsymbol{x}_{\mathcal{S}}\|_1 : \boldsymbol{x} \in \mathbb{R}^n : \|\boldsymbol{x}_{\mathcal{S}^c}\|_1 \leqslant \xi\|\boldsymbol{x}_{\mathcal{S}}\|_1\right\}. \tag{S.1}$$

Moreover, we say that $(\boldsymbol{M}, \mathcal{S}, \zeta)$ satisfies the compatibility condition if $\kappa(\boldsymbol{M}, \mathcal{S}, \zeta) > 0$.

The compatibility constant is related to ℓ_1-eigenvalue of $\boldsymbol{M}$ restricted to a cone in $\mathbb{R}^n$.

4.1 Proof of Proposition 1

The fact that $\|\widehat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1\|_1 = O_P(\xi|\mathcal{S}_0|)$ follows from Theorem 3 in Fan, Masini, and Medeiros (2021). We are left to show the second part. By the triangle inequality, for $t > T_0$:

$$
\begin{aligned}
|\widehat{\alpha}_t - \alpha_t - V_t| &= |(\widehat{\boldsymbol{\gamma}}_1 - \boldsymbol{\gamma}_1)'\boldsymbol{W}_{1t} + \widehat{\boldsymbol{\lambda}}_1'\widehat{\boldsymbol{F}}_t - \boldsymbol{\lambda}_1'\boldsymbol{F}_t + \widehat{\boldsymbol{\theta}}_1'\widehat{\boldsymbol{U}}_{-1t} - \boldsymbol{\theta}_1'\boldsymbol{U}_{-1t}| \\
&\leqslant |(\widehat{\boldsymbol{\gamma}}_1 - \boldsymbol{\gamma}_1)'\boldsymbol{W}_{1t}| + |\widehat{U}_{1t} - U_{1t}| + |\widehat{\boldsymbol{\theta}}_1'\widehat{\boldsymbol{U}}_{-1t} - \boldsymbol{\theta}_1'\boldsymbol{U}_{-1t}|.
\end{aligned}
$$

Using Hölder's inequality, the third term can be further bounded as

$$
\begin{aligned}
|\widehat{\boldsymbol{\theta}}_1'\widehat{\boldsymbol{U}}_{-1t} - \boldsymbol{\theta}_1'\boldsymbol{U}_{-1t}| &\leqslant |\widehat{\boldsymbol{\theta}}_1'(\widehat{\boldsymbol{U}}_{-1t} - \boldsymbol{U}_{-1t})| + |(\widehat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1)'\boldsymbol{U}_{-1t}| \\
&\leqslant \|\widehat{\boldsymbol{\theta}}_1\|_1\|\widehat{\boldsymbol{U}}_{-1t} - \boldsymbol{U}_{-1t}\|_\infty + \|\widehat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1\|_1\|\boldsymbol{U}_{-1t}\|_\infty \\
&\leqslant (\|\boldsymbol{\theta}_1\|_1 + \|\widehat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1\|_1)\|\widehat{\boldsymbol{U}}_{-1t} - \boldsymbol{U}_{-1t}\|_\infty + \|\widehat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1\|_1\|\boldsymbol{U}_{-1t}\|_\infty \\
&= O_P[(\|\boldsymbol{\theta}_1\|_1 + \upsilon|\mathcal{S}_0|\psi^{-1}(T))\upsilon + \upsilon|\mathcal{S}_0|\psi^{-1}(T)\psi^{-1}(n)].
\end{aligned}
$$

Combining the last two expressions we are left with

$$
|\widehat{\alpha}_t - \alpha_t - V_t| \leqslant |(\widehat{\boldsymbol{\gamma}}_1 - \boldsymbol{\gamma}_1)'\boldsymbol{W}_{1t}| + (1 + \|\boldsymbol{\theta}_1\|_1 + \|\widehat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1\|_1)\|\widehat{\boldsymbol{U}}_t - \boldsymbol{U}_t\|_\infty + \|\widehat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1\|_1\|\boldsymbol{U}_t\|_\infty.
$$

The first term is $O_P(1/\sqrt{T})$ by Assumption 3(a). The second is $O_P(|\mathcal{S}_0|\eta)$ because by Assumption 3(d) we have that $\|\boldsymbol{\theta}_1\|_1 \leqslant |\mathcal{S}_0|\|\boldsymbol{\theta}_1\|_\infty \leqslant C|\mathcal{S}_0|$ and $\|\widehat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1\|_1 = O_P(1)$ under the assumptions of the Proposition. Finally, the third term is $O_P(\xi|\mathcal{S}_0|n^{1/p})$ by Assumption 3(b) and the maximum inequality. Therefore we conclude that

$$
\widehat{\alpha}_t - \alpha_t - V_t = O_P\left(T^{-1/2} + |\mathcal{S}_0|\eta + \xi|\mathcal{S}_0|n^{1/p}\right) = O_P\left[|\mathcal{S}_0|(\eta + \xi n^{1/p})\right].
$$

Table S.1: **Results: Estimation and Inference (ArCo).**

The table reports estimation results using the ArCo methodology of Carvalho, Masini, and Medeiros (2018). In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a variety of different statistics. We consider the distribution over the treated municipalities. In Panel (a) we report the results for the R-squared of the pre-intervention model. Panel (b) displays the results for the average intervention effect over the experiment period (Δ). Panels (c) and (d) depict the results for the p-values of the ressampling test for the null hypothesis $\mathcal{H}_0 : \delta_t = 0, \forall t \in \{T_0 + 1, \ldots, T\}$ using respectively the test statistic $\phi(\widehat{\delta}_{T_0+1}, \ldots, \widehat{\delta}_T) = \sum_{t=T_0+1}^{T} \widehat{\delta}_t^2$ or $\phi(\widehat{\delta}_{T_0+1}, \ldots, \widehat{\delta}_T) = \sum_{t=T_0+1}^{T} |\widehat{\delta}_t|$. Finally, Panel (e) reports the results for the p-values for the test for the idiosyncratic contribution.

Panel (a): **R-squared**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0.1421	0.3367	0.4389	0.6276	0.7821	0.8958	0.4641	0.1981
II	0.4448	0.6555	0.8691	0.9218	0.9575	0.9851	0.9958	0.8899	0.1073
III	0.0639	0.3119	0.4957	0.6937	0.8181	0.9115	0.9679	0.6554	0.2018
IV	0.3688	0.6902	0.8823	0.9262	0.9635	0.9888	0.9987	0.8984	0.1056
V	0	0	0	0.0966	0.2210	0.4319	0.6975	0.1452	0.1545

Panel (b): **Average Treatment Effect (over time): Δ**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	-20.1194	-12.0679	-6.0420	-2.9966	-0.6335	1.7254	7.2911	-3.6588	4.3075
II	-40.6070	-25.4886	-9.9769	-3.1266	0.2057	9.9614	59.7638	-4.2132	11.6643
III	-37.8542	-8.5142	-3.3295	-1.0079	0.2364	3.7909	9.6714	-2.0799	5.8070
IV	-2.5440	-1.6212	-0.5723	0.1673	1.4634	3.8332	6.4165	0.4945	1.6339
V	-1.2218	-0.8548	-0.4922	-0.2797	0.0044	0.4945	1.1978	-0.2476	0.4234

Panel (c): **p-value of the test on squared values**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0.0085	0.1830	0.3702	0.6351	0.9147	0.9787	0.4077	0.2838
II	0	0.0388	0.2273	0.4876	0.7521	0.9521	1.0000	0.4905	0.2967
III	0	0.0306	0.2638	0.4894	0.6638	0.8928	0.9915	0.4735	0.2658
IV	0	0	0.0888	0.3802	0.7004	0.9029	0.9793	0.4092	0.3162
V	0	0.0894	0.3574	0.6936	0.9149	1.0000	1.0000	0.6452	0.3015

Panel (d): **p-value of the test on absolute values**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0	0.0787	0.3149	0.5691	0.8960	0.9872	0.3593	0.2995
II	0	0.0223	0.1818	0.5021	0.7273	0.9504	1.0000	0.4753	0.3095
III	0	0	0.2681	0.4532	0.6766	0.8655	1.0000	0.4624	0.2671
IV	0	0	0.1033	0.3946	0.7066	0.9318	0.9876	0.4124	0.3220
V	0	0.1234	0.3787	0.6745	0.8809	0.9957	1.0000	0.6284	0.2886

Table S.2: **Results: Elasticities and Optimal Prices (ArCo).**

The table reports elasticities estimates as well the percentage difference between the current prices and the optimal price maximizing profit when the ArCo methodology by Carvalho, Masini, and Medeiros (2018) is used. In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a given statistic. We consider the distribution over the selected treated municipalities. **We only report results concerning the cities where the estimated Δ has the correct sign and the effects are statistical significance at the 10% level**. The last column indicates the fraction of cities that satisfy the criterium described above. In Panel (a) we report the results for the estimated elasticities. In Panel (b) we show the results for the difference between the current price and the optimal price.

Panel (a): **Elasticities**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev	Fraction
I	-6.1256	-6.0847	-3.4902	-2.7145	-2.1592	-1.3585	-1.2700	-3.1159	1.4785	0.1443
II	-16.8229	-16.8229	-12.6336	-11.4970	-7.5925	-4.0746	-4.0746	-10.5119	3.9061	0.0882
III	-3.0759	-3.0759	-2.8387	-2.0602	-1.8896	-1.6480	-1.6480	-2.2876	0.5477	0.0755
IV	-44.2416	-34.5020	-11.9050	-6.5419	-4.5606	-2.4450	-1.9634	-10.6109	10.1396	0.2400
V	-135.5289	-135.5289	-19.1707	-10.1509	-5.4511	-4.3575	-4.3575	-30.8017	51.5716	0.0545

Panel (b): **Price Discrepancies (% Difference)**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev	Fraction
I	-15.5005	-15.4441	-9.3372	-5.2233	-0.5061	13.6720	15.7075	-4.2981	8.5470	0.1443
II	-21.3235	-21.3235	-20.3202	-19.9466	-17.6746	-12.0246	-12.0246	-18.6809	2.8620	0.0882
III	-9.0999	-9.0999	-7.7113	-1.0822	1.1540	4.9837	4.9837	-2.4244	5.1882	0.0755
IV	-18.5830	-18.2201	-15.5075	-12.0222	-8.7234	1.1269	5.7524	-11.2130	5.9686	0.2400
V	-19.3704	-19.3704	-17.1312	-14.8087	-10.5669	-8.2649	-8.2649	-14.1585	4.1117	0.0545

Table S.3: **Results: Estimation and Inference (PCR).**

The table reports estimation results using principal component regressions. In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a variety of different statistics. We consider the distribution over the treated municipalities. In Panel (a) we report the results for the R-squared of the pre-intervention model. Panel (b) displays the results for the average intervention effect over the experiment period (Δ). Panels (c) and (d) depict the results for the p-values of the ressampling test for the null hypothesis $\mathcal{H}_0 : \delta_t = 0, \forall t \in \{T_0+1,\ldots,T\}$ using respectively the test statistic $\phi(\widehat{\delta}_{T_0+1},\ldots,\widehat{\delta}_T) = \sum_{t=T_0+1}^{T} \widehat{\delta}_t^2$ or $\phi(\widehat{\delta}_{T_0+1},\ldots,\widehat{\delta}_T) = \sum_{t=T_0+1}^{T} |\widehat{\delta}_t|$.

Panel (a): **R-squared**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0.1115	0.1727	0.3307	0.4491	0.6011	0.7294	0.7892	0.4517	0.1707
II	0.2549	0.4633	0.7428	0.8345	0.8815	0.9456	0.9759	0.7898	0.1445
III	0.1026	0.1588	0.2489	0.3466	0.5095	0.6296	0.6944	0.3751	0.1545
IV	0.1300	0.2384	0.5805	0.7173	0.8236	0.8941	0.9627	0.6723	0.1996
V	0.0255	0.0366	0.0739	0.1236	0.2068	0.3815	0.5079	0.1545	0.1033

Panel (b): **Average Treatment Effect (over time): Δ**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	-21.9722	-17.1898	-7.6521	-3.4870	-1.0735	1.6398	3.6122	-5.0798	5.6688
II	-47.0186	-32.5355	-15.2901	-7.5150	-2.8772	9.9514	40.2040	-9.2082	12.9511
III	-55.4751	-17.1204	-7.2165	-3.4482	-0.6900	1.8316	8.8381	-5.6288	9.8650
IV	-4.3269	-1.9948	-0.7039	0.2394	1.5064	4.1167	7.3901	0.5691	1.9752
V	-2.0826	-0.9796	-0.5058	-0.1766	0.1292	0.6744	1.6705	-0.1831	0.5190

Panel (c): ***p*-value of the test on squared values**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0	0.0723	0.2553	0.6170	0.8985	0.9915	0.3445	0.3063
II	0.0289	0.0421	0.2355	0.4566	0.6901	0.8983	0.9752	0.4697	0.2844
III	0	0.0664	0.2809	0.4511	0.6340	0.9336	1.0000	0.4624	0.2478
IV	0	0.0723	0.2459	0.4153	0.7169	0.9917	1.0000	0.4794	0.2878
V	0	0.0596	0.3277	0.6511	0.9319	1.0000	1.0000	0.6050	0.3302

Panel (d): ***p*-value of the test on absolute values**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0	0.0511	0.2128	0.6053	0.9019	0.9957	0.3199	0.3199
II	0	0.0207	0.1570	0.4256	0.6942	0.9298	0.9628	0.4481	0.3038
III	0	0.0102	0.2128	0.4128	0.5957	0.9319	1.0000	0.4248	0.2689
IV	0	0.0517	0.2149	0.4070	0.7521	0.9690	1.0000	0.4710	0.3001
V	0	0.0511	0.2681	0.6638	0.9234	1.0000	1.0000	0.6084	0.3362

Table S.4: **Results: Elasticities and Optimal Prices (PCR).**

The table reports elasticities estimates as well the percentage difference between the current prices and the optimal price maximizing profit when the counterfactuals are estimated by principal component regression. In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a given statistic. We consider the distribution over the selected treated municipalities. **We only report results concerning the cities where the estimated Δ has the correct sign and the effects are statistical significance at the 10% level**. The last column indicates the fraction of cities that satisfy the criterium described above. In Panel (a) we report the results for the estimated elasticities. In Panel (b) we show the results for the difference between the current price and the optimal price.

Panel (a): **Elasticities**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev	Fraction
I	-6.5287	-5.5838	-4.3723	-3.5023	-2.9266	-1.7697	-0.9696	-3.5689	1.1443	0.2784
II	-17.7671	-17.5199	-14.5000	-13.1484	-8.6987	-2.8126	-1.9998	-11.8098	4.2565	0.1275
III	-3.3805	-3.3805	-3.2669	-2.9047	-2.7249	-2.3503	-2.3503	-2.9405	0.3565	0.0755
IV	-15.8735	-15.8735	-12.4477	-11.0990	-9.2416	-1.0297	-1.0297	-10.3376	4.6432	0.0700
V	-36.2752	-36.2752	-25.0377	-15.3515	-5.8105	-3.3284	-3.3284	-16.8591	12.4347	0.0545

Panel (b): **Price Discrepancies (% Difference)**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev	Fraction
I	-16.0044	-14.6686	-12.1884	-9.3865	-6.5784	6.3136	27.9067	-7.6159	8.1823	0.2784
II	-21.4814	-21.4382	-20.8472	-20.4928	-18.5425	-2.0321	0.7073	-18.4545	5.8816	0.1275
III	-10.5643	-10.5643	-10.0456	-8.1187	-6.9926	-4.0815	-4.0815	-8.1200	2.1928	0.0755
IV	-16.5633	-16.5633	-15.6963	-15.2083	-14.2557	28.8451	28.8451	-9.0521	16.7294	0.0700
V	-18.3610	-18.3610	-17.7424	-16.3414	-11.1343	-4.7169	-4.7169	-14.1062	5.2776	0.0545

Table S.5: **Results: Estimation and Inference (no trend).**

The table reports estimation results without the trend component in the counterfactual model. In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a variety of different statistics. We consider the distribution over the treated municipalities aggregated at the state level. In Panel (a) we report the results for the R-squared of the pre-intervention model. Panel (b) displays the results for the average intervention effect over the experiment period (Δ). Panels (c) and (d) depict the results for the p-values of the ressampling test for the null hypothesis $\mathcal{H}_0 : \delta_t = 0, \forall t \in \{T_0 + 1, \dots, T\}$ using respectively the test statistic $\phi(\widehat{\delta}_{T_0+1}, \dots, \widehat{\delta}_T) = \sum_{t=T_0+1}^{T} \widehat{\delta}_t^2$ or $\phi(\widehat{\delta}_{T_0+1}, \dots, \widehat{\delta}_T) = \sum_{t=T_0+1}^{T} |\widehat{\delta}_t|$. Finally, Panel (e) reports the results for the p-values for the test for the idiosyncratic contribution.

Panel (a): **R-squared**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0.1112	0.1983	0.3463	0.4910	0.6302	0.7556	0.9029	0.4869	0.1777
II	0.4876	0.6913	0.8721	0.9280	0.9563	0.9850	0.9945	0.9007	0.0905
III	0.1141	0.2904	0.5243	0.7085	0.8324	0.9336	0.9600	0.6736	0.2041
IV	0.3824	0.6693	0.8802	0.9344	0.9632	0.9869	0.9986	0.8969	0.1101
V	0.0243	0.0378	0.0895	0.1461	0.2706	0.4143	0.6396	0.1876	0.1321

Panel (b): **Average Treatment Effect (over time): Δ**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	-16.1305	-11.3082	-5.0625	-2.6195	-0.8542	1.4314	9.9071	-3.3187	4.1600
II	-46.3695	-27.3151	-10.4665	-4.1799	-0.6947	6.8649	58.5092	-5.9179	11.9431
III	-26.5438	-9.0437	-3.0657	-0.9038	0.6108	4.8286	16.0986	-1.5804	5.2233
IV	-3.9357	-1.6404	-0.5186	0.2410	1.2506	4.0381	6.3938	0.5208	1.7143
V	-1.0360	-0.5738	-0.2827	-0.1076	0.1859	0.7279	1.0478	-0.0468	0.3770

Panel (c): **p-value of the test on squared values**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0.0143	0.1564	0.4213	0.6298	0.8764	0.9872	0.4112	0.2786
II	0	0.0198	0.1818	0.4628	0.7273	0.9793	1.0000	0.4626	0.3138
III	0	0.0170	0.2638	0.4745	0.7064	0.9583	1.0000	0.4864	0.2839
IV	0	0	0.1302	0.3802	0.7025	0.9545	0.9876	0.4074	0.3110
V	0	0.0766	0.3447	0.8170	0.9872	1.0000	1.0000	0.6779	0.3238

Panel (d): **p-value of the test on absolute values**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0	0.1000	0.4000	0.5936	0.8979	0.9957	0.3785	0.2885
II	0	0.0025	0.1446	0.4029	0.7355	0.9694	1.0000	0.4471	0.3213
III	0	0	0.2170	0.4787	0.7234	0.9149	1.0000	0.4757	0.2918
IV	0	0	0.0992	0.3616	0.7066	0.9360	0.9917	0.4000	0.3137
V	0	0.1064	0.4340	0.8021	0.9915	1.0000	1.0000	0.6974	0.3104

Panel (e): **p-value of the test for idiosyncratic contribution**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0	0.0300	0.0820	0.2525	0.6636	0.9500	0.1824	0.2145
II	0.0080	0.0224	0.0620	0.1250	0.2520	0.4940	0.6760	0.1771	0.1488
III	0	0	0.0080	0.0590	0.1540	0.3468	0.5460	0.0995	0.1180
IV	0.0300	0.0470	0.0990	0.1920	0.2800	0.4410	0.6400	0.2093	0.1317
V	0	0.0240	0.1160	0.2890	0.4180	0.7080	0.8400	0.2977	0.2083

Table S.6: **Results: Elasticities and Optimal Prices (no trend).**

The table reports elasticities estimates as well the percentage difference between the current prices and the optimal price maximizing profit. In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a given statistic. We consider the distribution over the selected treated municipalities. **We only report results concerning the cities where the estimated Δ has the correct sign and the effects are statistical significance at the 10% level**. The last column indicates the fraction of cities that satisfy the criterium described above. In Panel (a) we report the results for the estimated elasticities. In Panel (b) we show the results for the difference between the current price and the optimal price.

Panel (a): **Elasticities**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev	Fraction
I	-6.1709	-6.1363	-4.3408	-2.9859	-2.2372	-1.5650	-1.4268	-3.4141	1.4124	0.1753
II	-17.2147	-16.8507	-12.9427	-11.9334	-8.7978	-3.4640	-2.8945	-10.9383	3.8642	0.1569
III	-2.8147	-2.8147	-2.4532	-1.8840	-1.6626	-1.6254	-1.6254	-2.0550	0.4905	0.0755
IV	-32.7958	-24.6827	-11.4079	-6.6815	-3.9395	-2.6159	-2.4158	-8.5285	7.1821	0.2000
V	-30.6356	-30.6356	-28.5022	-25.0506	-23.6188	-15.6706	-15.6706	-24.7547	5.2214	0.0545

Panel (b): **Price Discrepancies (% Difference)**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev	Fraction
I	-15.5604	-15.5143	-12.1436	-6.9179	-1.3130	8.7227	11.3806	-6.3656	7.4661	0.1753
II	-21.3911	-21.3250	-20.4324	-20.1047	-18.5991	-9.0741	-7.0214	-18.6833	3.6201	0.1569
III	-7.5915	-7.5915	-4.5816	1.2230	4.7269	5.4073	5.4073	0.0690	5.2832	0.0755
IV	-18.1886	-17.4421	-15.3303	-12.2240	-7.0189	-0.4867	0.9835	-10.4346	5.6478	0.2000
V	-18.1073	-18.1073	-17.9851	-17.7375	-17.6224	-16.5487	-16.5487	-17.6231	0.5603	0.0545

Table S.7: **Results: Estimation and Inference (state level).**

The table reports estimation results at the state level. In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a variety of different statistics. We consider the distribution over the treated municipalities aggregated at the state level. In Panel (a) we report the results for the R-squared of the pre-intervention model. Panel (b) displays the results for the average intervention effect over the experiment period (Δ). Panels (c) and (d) depict the results for the p-values of the ressampling test for the null hypothesis $\mathcal{H}_0 : \delta_t = 0, \forall t \in \{T_0+1, \ldots, T\}$ using respectively the test statistic $\phi(\widehat{\delta}_{T_0+1}, \ldots, \widehat{\delta}_T) = \sum_{t=T_0+1}^{T} \widehat{\delta}_t^2$ or $\phi(\widehat{\delta}_{T_0+1}, \ldots, \widehat{\delta}_T) = \sum_{t=T_0+1}^{T} |\widehat{\delta}_t|$. Finally, Panel (e) reports the results for the p-values for the test for the idiosyncratic contribution.

Panel (a): **R-squared**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0.3553	0.3804	0.6776	0.8027	0.8969	0.9573	0.9603	0.7593	0.1814
II	0.8830	0.8895	0.9410	0.9812	0.9934	0.9962	0.9962	0.9659	0.0351
III	0.2983	0.3604	0.6552	0.7726	0.8651	0.9422	0.9642	0.7329	0.1763
IV	0.7566	0.8014	0.9377	0.9684	0.9874	0.9952	0.9962	0.9480	0.0587
V	0.0996	0.1249	0.1795	0.3048	0.5028	0.8898	0.9024	0.3687	0.2298

Panel (b): **Average Treatment Effect (over time): Δ**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	-4.9172	-4.6035	-3.3475	-1.8782	0.1261	1.8318	1.8886	-1.6650	2.1306
II	-17.5812	-16.8032	-9.1995	-2.6898	2.0167	13.5124	14.5276	-2.9574	9.0027
III	-7.5112	-6.7144	-3.5990	-0.9333	0.3148	14.7754	32.3728	-0.2965	7.6855
IV	-2.0756	-1.7365	-0.7512	-0.3154	0.3816	0.8821	1.0061	-0.2839	0.7757
V	-0.7695	-0.6821	-0.3421	-0.1571	-0.0078	0.3294	0.4216	-0.1837	0.2904

Panel (c): **p-value of the test on squared values**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0	0.0511	0.2511	0.4883	0.8736	0.9957	0.3041	0.2823
II	0	0	0.0548	0.3017	0.5610	0.9731	0.9876	0.3717	0.3427
III	0	0.0194	0.2043	0.4128	0.7511	0.9387	0.9830	0.4503	0.3201
IV	0.0331	0.0793	0.2500	0.4215	0.5723	0.7901	0.8017	0.4166	0.2206
V	0	0.0070	0.3170	0.8426	0.9543	0.9930	1.0000	0.6470	0.3741

Panel (d): **p-value of the test on absolute values**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0	0.0032	0.3404	0.4553	0.9109	0.9957	0.3189	0.2930
II	0	0	0.0207	0.3430	0.4824	0.9599	0.9628	0.3493	0.3421
III	0	0	0.1372	0.3830	0.7660	0.9257	0.9617	0.4213	0.3406
IV	0.0331	0.0605	0.2397	0.4793	0.5981	0.7837	0.8140	0.4256	0.2377
V	0	0	0.3319	0.7830	0.9340	0.9930	1.0000	0.6237	0.3796

Panel (e): **p-value of the test for idiosyncratic contribution**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	0	0	0	0.0020	0.0210	0.1724	0.1920	0.0314	0.0593
II	0	0	0.0110	0.0380	0.0545	0.2964	0.3440	0.0616	0.0893
III	0	0	0.0105	0.0420	0.0990	0.5157	0.6080	0.1017	0.1640
IV	0.0940	0.1164	0.1780	0.2040	0.2310	0.5657	0.6420	0.2408	0.1292
V	0	0	0.0350	0.0820	0.1350	0.5735	0.6780	0.1370	0.1767

Table S.8: **Results: Elasticities and Optimal Prices (state level).**

The table reports elasticities estimates as well the percentage difference between the current prices and the optimal price maximizing profit. In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a given statistic. We consider the distribution over the selected treated municipalities. **We only report results concerning the cities where the estimated Δ has the correct sign and the effects are statistical significance at the 10% level**. The last column indicates the fraction of cities that satisfy the criterium described above. In Panel (a) we report the results for the estimated elasticities. In Panel (b) we show the results for the difference between the current price and the optimal price.

Panel (a): **Elasticities**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev	Fraction
I	-1.8282	-1.8282	-1.7647	-1.5235	-1.1270	-0.8921	-0.8921	-1.4431	0.3720	0.2222
II	-11.5235	-11.5235	-7.7493	-5.8728	-4.4528	-4.1237	-4.1237	-6.5147	2.9592	0.1852
III	-3.2089	-3.2089	-2.9427	-1.9333	-1.0804	-0.9708	-0.9708	-2.0116	1.1006	0.1481
IV	–	–	–	–	–	–	–	–	–	–
V	–	–	–	–	–	–	–	–	–	–

Panel (b): **Price Discrepancies (% Difference)**

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev	Fraction
I	3.6865	3.6865	4.6703	9.3255	20.7040	32.3850	32.3850	13.3495	11.2129	0.2222
II	-19.9566	-19.9566	-17.4338	-15.7817	-13.0450	-12.1705	-12.1705	-15.5676	3.0364	0.1852
III	-9.7734	-9.7734	-8.2241	4.9918	21.4037	26.1490	26.1490	6.5898	17.5845	0.1481
IV	–	–	–	–	–	–	–	–	–	–
V	–	–	–	–	–	–	–	–	–	–

Table S.9: **Results: Estimation and Inference (Before-and-After).**

The table reports estimation the average treatment effect using the before-and-after estimator. In each panel we report, for each product, the minimum, the 5%-, 25%-, 50%-, 75%-, and 95%-quantiles, maximum, average, and standard deviation for a variety of different statistics. We consider the distribution over the treated municipalities.

Product	Min	5%-quantile	25%-quantile	Median	75%-quantile	95% quantile	Max	Average	Std. Dev
I	-23.8652	-17.2270	-8.1333	-4.1126	-1.1093	2.1760	11.5150	-5.2622	6.0399
II	-74.8229	-53.2274	-30.7149	-18.4681	-10.3370	1.8621	13.1138	-22.0736	16.9785
III	-48.8512	-15.3860	-5.6494	-2.1679	-0.5397	2.4336	11.1025	-3.9888	7.0311
IV	-5.5069	-4.7638	-2.1703	-1.2016	-0.1093	1.7353	3.5901	-1.2483	1.9274
V	-2.0595	-1.3942	-0.8139	-0.4505	-0.1244	0.4032	1.1809	-0.4682	0.5426

Figure S.1: Data for Product II.

Panel (a) reports the daily sales divided by the number of stores aggregated for all cities as well as for the treatment and control groups. The plot also indicates the date of the intervention. Panels (b) and (c) display the distribution of the average sales per store over time across municipalities in the treatment and control groups, respectively. Panels (d) and (e) present fan plots of sales across municipalities in the treatment and control groups for each given time point. The black curves represent the cross-sectional mean over time and the vertical green line indicates the date of intervention.

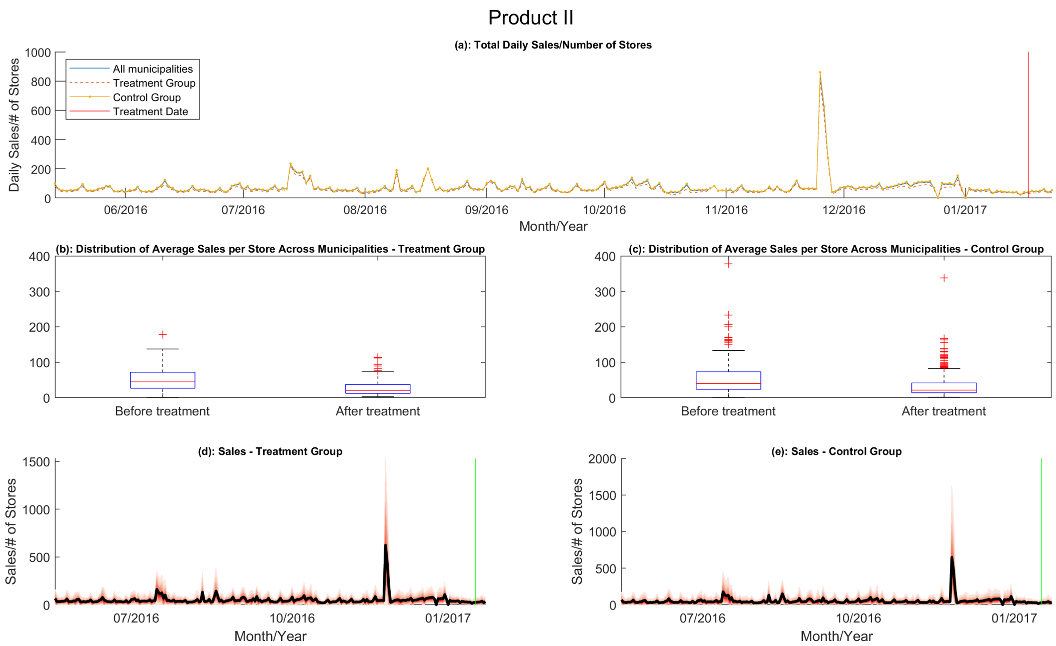


Figure S.2: Data for Product III.

Panel (a) reports the daily sales divided by the number of stores aggregated for all cities as well as for the treatment and control groups. The plot also indicates the date of the intervention. Panels (b) and (c) display the distribution of the average sales per store over time across municipalities in the treatment and control groups, respectively. Panels (d) and (e) present fan plots of sales across municipalities in the treatment and control groups for each given time point. The black curves represent the cross-sectional mean over time and the vertical green line indicates the date of intervention.

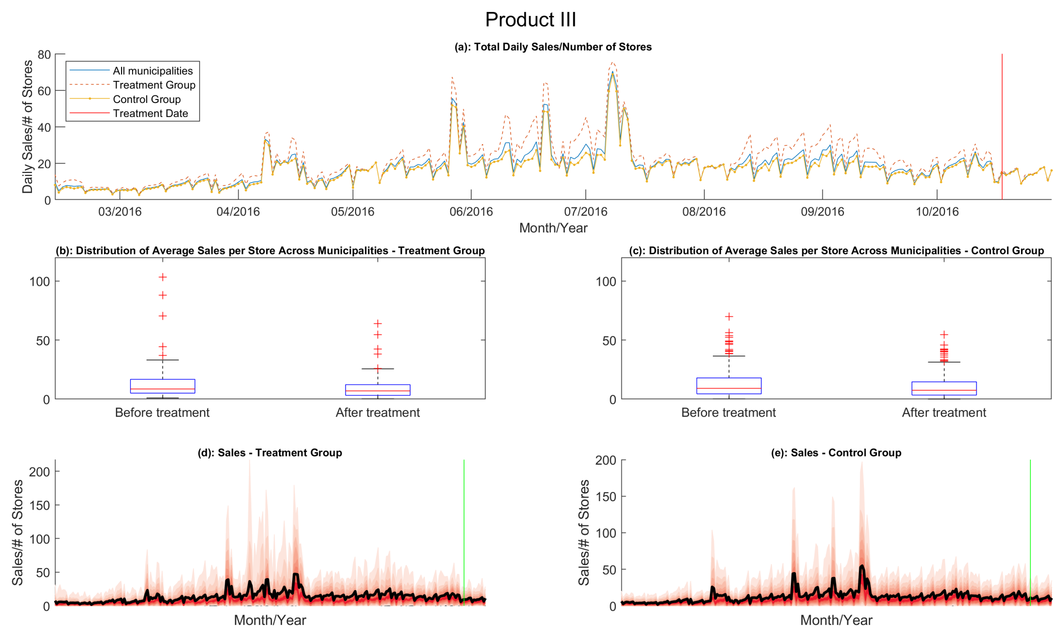


Figure S.3: Data for Product IV.

Panel (a) reports the daily sales divided by the number of stores aggregated for all cities as well as for the treatment and control groups. The plot also indicates the date of the intervention. Panels (b) and (c) display the distribution of the average sales per store over time across municipalities in the treatment and control groups, respectively. Panels (d) and (e) present fan plots of sales across municipalities in the treatment and control groups for each given time point. The black curves represent the cross-sectional mean over time and the vertical green line indicates the date of intervention.

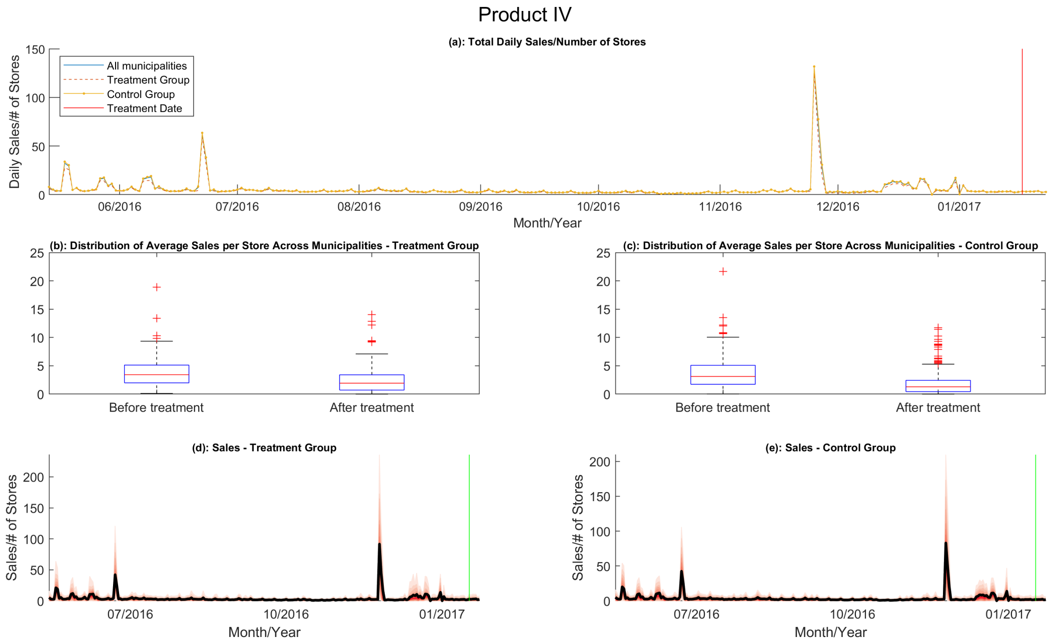


Figure S.4: Data for Product V.

Panel (a) reports the daily sales divided by the number of stores aggregated for all cities as well as for the treatment and control groups. The plot also indicates the date of the intervention. Panels (b) and (c) display the distribution of the average sales per store over time across municipalities in the treatment and control groups, respectively. Panels (d) and (e) present fan plots of sales across municipalities in the treatment and control groups for each given time point. The black curves represent the cross-sectional mean over time and the vertical green line indicates the date of intervention.

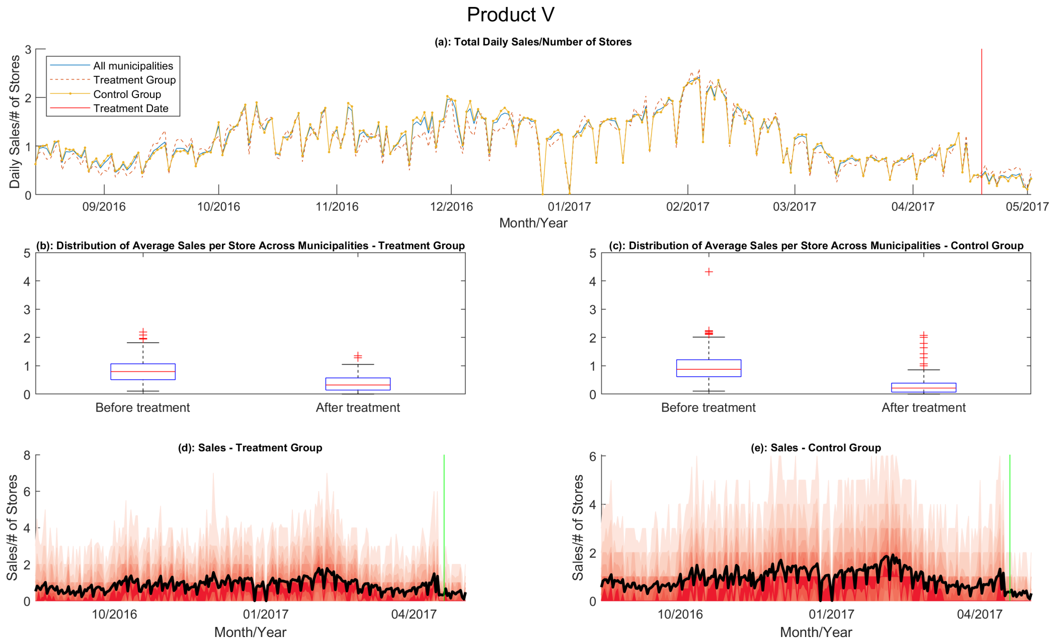


Figure S.5: Results for Product II

Panel (a) displays a fan plot, across n_1 municipalities in the treatment group, of the p-values of the re-sampling test for the null $\mathscr{H}_0 : \delta_t = 0$ at each time t after the treatment. The black curve represents the median p-value across municipalities over t. Panel (b) shows an example for one municipality. The panel depicts the actual and counterfactual sales per store for the post-treatment period. 95% confidence intervals for the counterfactual path is also displayed.

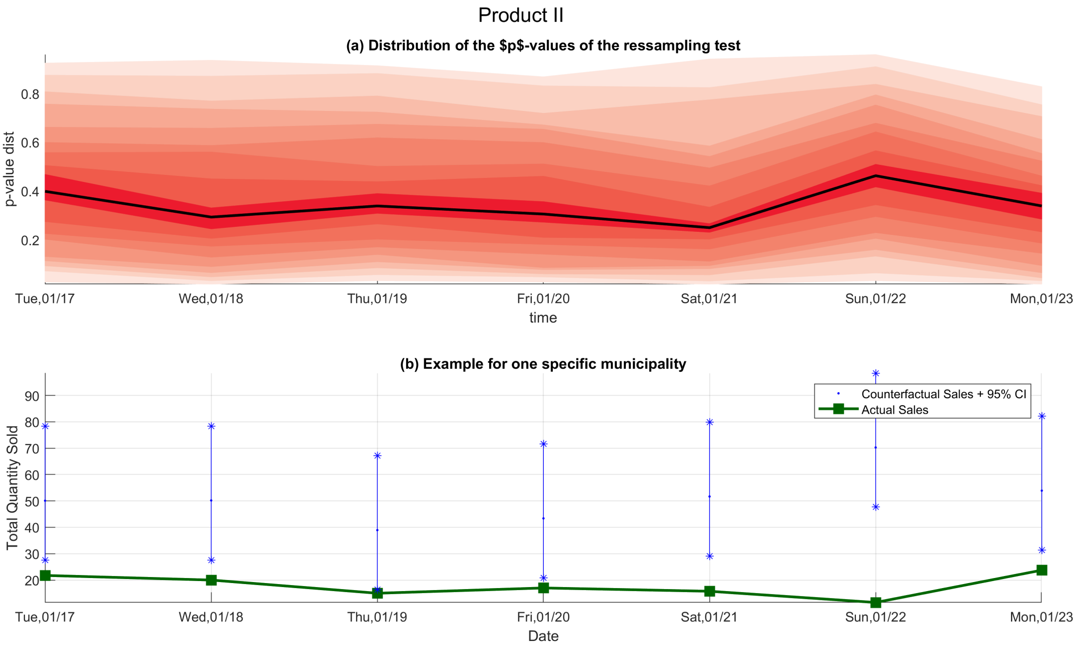


Figure S.6: Results for Product III

Panel (a) displays a fan plot, across n_1 municipalities in the treatment group, of the p-values of the re-sampling test for the null $\mathscr{H}_0 : \delta_t = 0$ at each time t after the treatment. The black curve represents the median p-value across municipalities over t. Panel (b) shows an example for one municipality. The panel depicts the actual and counterfactual sales per store for the post-treatment period. 95% confidence intervals for the counterfactual path is also displayed.

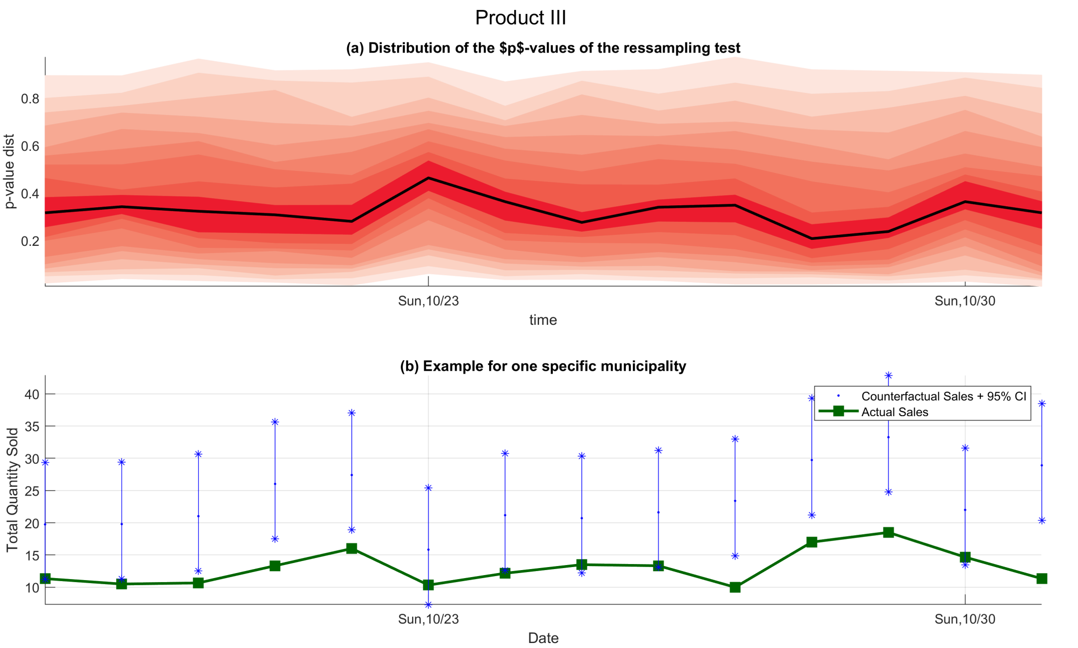


Figure S.7: Results for Product IV

Panel (a) displays a fan plot, across n_1 municipalities in the treatment group, of the p-values of the re-sampling test for the null $\mathscr{H}_0 : \delta_t = 0$ at each time t after the treatment. The black curve represents the median p-value across municipalities over t. Panel (b) shows an example for one municipality. The panel depicts the actual and counterfactual sales per store for the post-treatment period. 95% confidence intervals for the counterfactual path is also displayed.

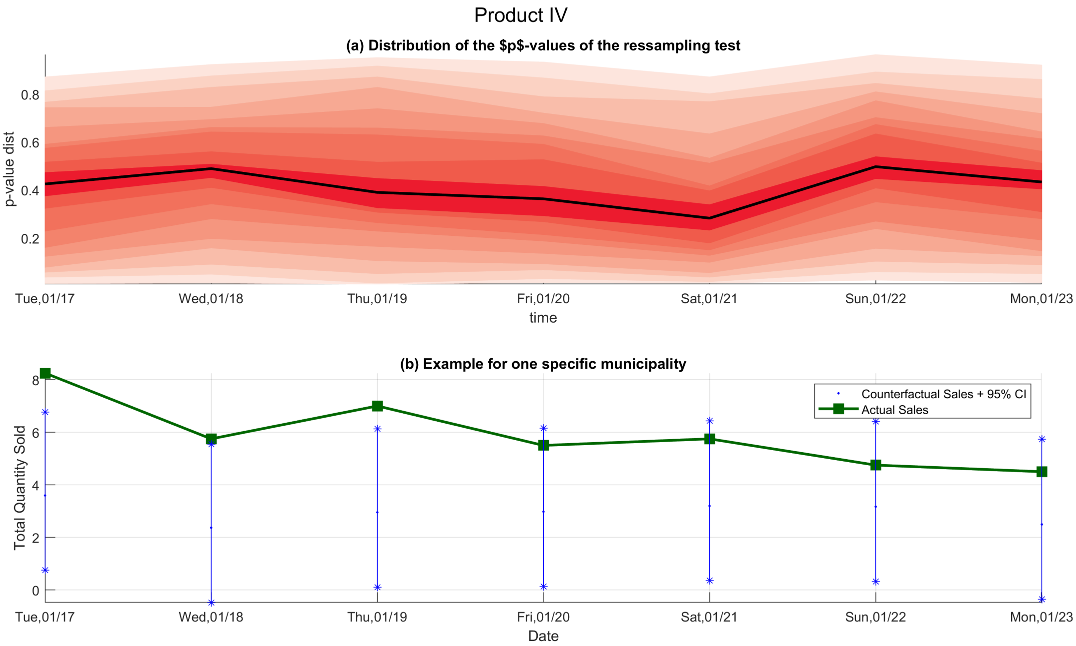


Figure S.8: Results for Product V

Panel (a) displays a fan plot, across n_1 municipalities in the treatment group, of the p-values of the re-sampling test for the null $\mathscr{H}_0 : \delta_t = 0$ at each time t after the treatment. The black curve represents the median p-value across municipalities over t. Panel (b) shows an example for one municipality. The panel depicts the actual and counterfactual sales per store for the post-treatment period. 95% confidence intervals for the counterfactual path is also displayed.

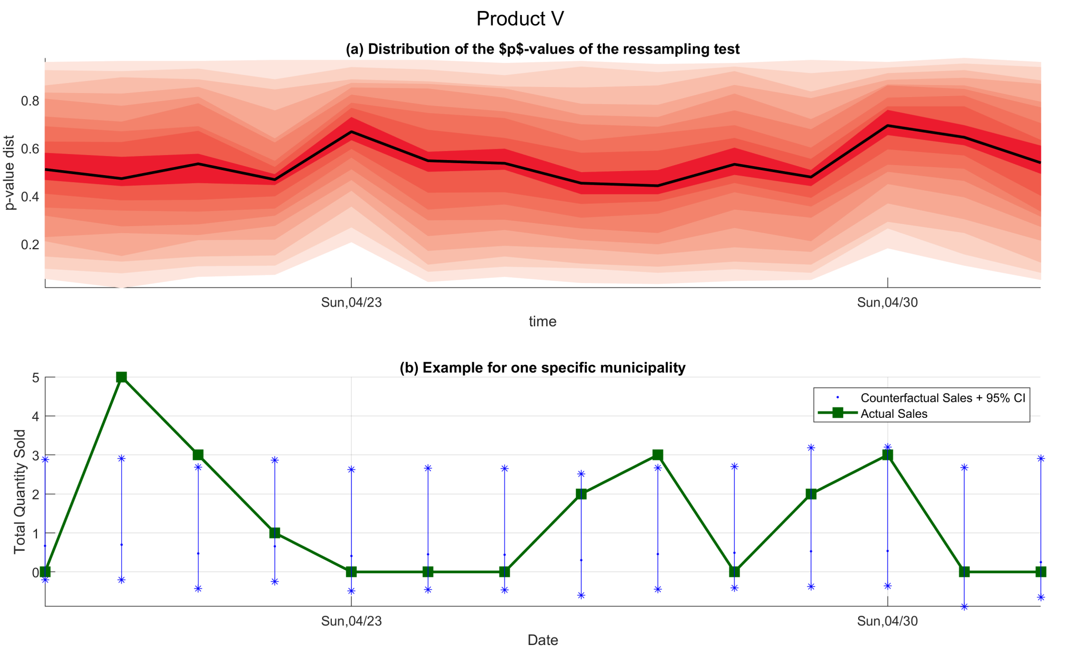
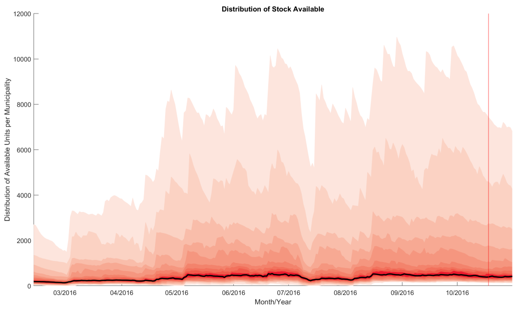
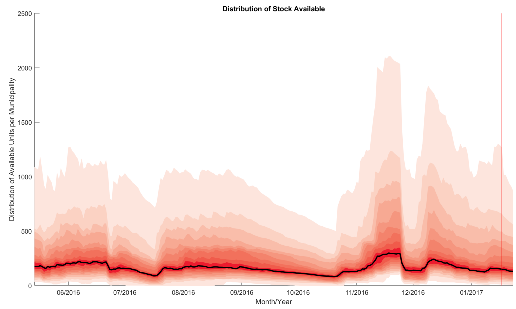


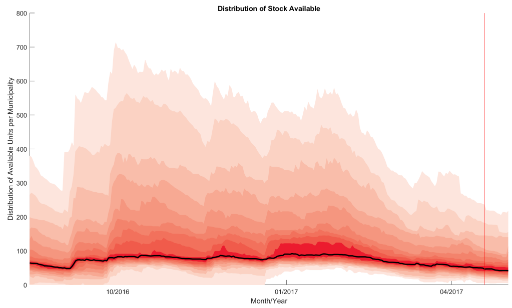
Figure S.9: Daily Inventory Distribution.



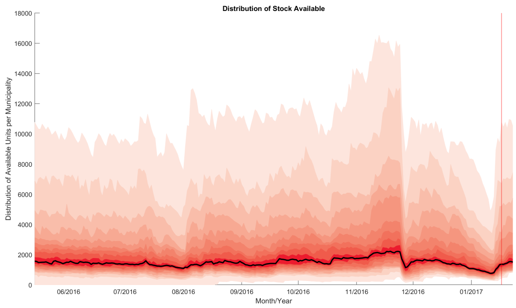
Product I



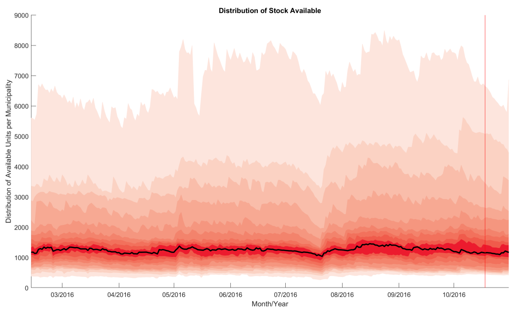
Product II



Product III



Product IV



Product V

References

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