

# Detecting small shift on the mean by finite moving average\*

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## Abstract

We consider a sequence of independent Gaussian random variables whose mean remains constant until an unknown time  $r$  when it increases of  $\delta$ , and then remains constant afterwards. We want to detect the abrupt change as soon as possible. We propose a new study of detection rules based on finite moving averages.

Finite moving averages (FMA) depend on two parameters: the window size  $A$  and a threshold  $h$ . We study the asymptotic properties of finite moving averages detection rules for a fixed threshold  $h$  less than the assumed value of the shift  $\delta$ , as the window size  $A$  is varying. For the stationary average delay time (SADT), the first order term of the asymptotic development with respect to the average run length (ARL) is the same (up to  $\epsilon$ ) than for the CUSUM, Shiryaev–Roberts or GLR detection rules, and the second term plays in favour of the finite moving average, as for exponentially weighted moving average (EWMA).

Numerical simulations confirm that finite moving average detection rules are better than GLR procedure for detecting small shifts and they show no real difference with the CUSUM procedure.

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