

STA135 HW5 Solution

7.3

$$Y = Z\beta + \varepsilon$$

$$V^{-1/2}Y = V^{-1/2}Z\beta + V^{-1/2}\varepsilon$$

$$\text{Let } Y^* = V^{-1/2}Y, Z^* = V^{-1/2}Z \text{ and } \varepsilon^* = V^{-1/2}\varepsilon$$

Now, we can apply LSE to $Y^* = Z^*\beta + \varepsilon^*$ since $\text{cov}(\varepsilon^*) = \sigma^2 I$.

$$\text{Thus, } \hat{\beta} = [(Z^*)'Z^*]^{-1}(Z^*)'Y^* = (Z'V^{-1}Z)Z'V^{-1}Y$$

9.1

$$\text{Now } L' = (.9 \ .7 \ .5), \text{ and } LL' = \begin{pmatrix} 0.8100 & 0.6300 & 0.4500 \\ 0.6300 & 0.4900 & 0.3500 \\ 0.4500 & 0.3500 & 0.2500 \end{pmatrix}.$$

$$\text{Thus, } LL' + \Psi = \begin{pmatrix} 0.8100 + 0.19 & 0.6300 & 0.4500 \\ 0.6300 & 0.4900 + 0.51 & 0.3500 \\ 0.4500 & 0.3500 & 0.2500 + 0.75 \end{pmatrix} = \rho$$

9.2

(a)

$$h_1^2 = .81, h_2^2 = .49, h_3^2 = .25$$

The portion of the 1st variable contributed by one common factor is 0.81. The portion of the 2nd variable contributed by one common factor is 0.49. The portion of the 3rd variable contributed by one common factor is 0.25.

(b)

$$\text{Corr}(Z_1, F_1) = 0.9, \text{Corr}(Z_2, F_1) = 0.7, \text{Corr}(Z_3, F_1) = 0.5$$

Z_1 carries the greatest weight since the correlation between Z_1 and F_1 is the largest.

9.26

(a)

$$m=1: F = \left(\sqrt{\hat{\lambda}_1} \hat{e}_1 \right) = \sqrt{3779} \begin{pmatrix} 0.4541 \\ 0.4941 \\ 0.5123 \\ 0.5359 \end{pmatrix} = \begin{pmatrix} 27.9178 \\ 30.3757 \\ 31.4930 \\ 32.9410 \end{pmatrix}$$

$$m=2: F = \left(\sqrt{\hat{\lambda}_1} \hat{e}_1; \sqrt{\hat{\lambda}_2} \hat{e}_2 \right) = \begin{pmatrix} 27.9178 & 7.9115 \\ 30.3757 & -12.9347 \\ 31.4930 & 1.0622 \\ 32.9410 & 4.2069 \end{pmatrix}$$

(b)

$$S_n = \frac{150 - 1}{150} S = 10^3 \begin{pmatrix} 1.0811 & 0.8257 & 0.7581 & 0.7789 \\ 0.8257 & 1.1209 & 0.8446 & 0.9206 \\ 0.7581 & 0.8446 & 1.3272 & 0.8985 \\ 0.7789 & 0.9206 & 0.8985 & 1.3858 \end{pmatrix}$$
$$m=1: F = \begin{pmatrix} 0.8133 \\ 0.9083 \\ 0.7779 \\ 0.8130 \end{pmatrix}$$

For the case $m=2$, since $(p - m)^2 - (p + m) = (4 - 2)^2 - (4 + 2) < 0$, we can not perform MLE to estimate the loading matrix.

(c)

part(a):

$$m=1: F^* = \begin{pmatrix} 27.9178 \\ 30.3757 \\ 31.4930 \\ 32.9410 \end{pmatrix}$$
$$m=2: F^* = \begin{pmatrix} 26.4967 & -11.8288 \\ 15.0169 & -29.4021 \\ 24.8487 & -19.3773 \\ 27.9762 & -17.8924 \end{pmatrix}$$

part(b):

$$m=1: F^* = \begin{pmatrix} 0.8133 \\ 0.9083 \\ 0.7779 \\ 0.8130 \end{pmatrix}$$

For the case $m=2$, since $(p - m)^2 - (p + m) = (4 - 2)^2 - (4 + 2) < 0$, we can not perform MLE to estimate the loading matrix.