

**PROBABILITY AND STATISTICS
THROUGHOUT THE CENTURIES**

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Outline of the evolution of probability and statistics throughout the centuries, starting with classical antiquity, passing to the Hellenistic period (323-146 B.C.), jumping to the aftermath of the Renaissance in Western Europe (*roughly between the 14th and the 16th centuries*), and concluding with the 19th and the 20th centuries, as well as the contemporary period.

TOPICS TO BE DISCUSSED

1. What is probability.
2. What is statistics.
3. Relation between probability and statistics.
4. Classical antiquity-Hellenistic period.
5. Probability in the 16th and the 17th centuries.
6. Probability in the 18th century.
7. Statistics in the 19th and the 20th centuries.
8. Probability in the 20th century.
9. Statistics and probability in our times-Their influence to sciences.
10. Closing remarks.

This was the topic of the inaugural address of this author as Corresponding Member of the Academy of Athens on November 10, 2009, and a version of it is published in Greek in the Proceedings of the Academy of Athens, t. 84 A (2009).

1. What is probability

Probability is the formulation of uncertainty in mathematical terms, which uncertainty exists because of ignorance or complexity.

Probability is interested in what is empirically possible and not only logically possible.

Thus, probability encompasses knowledge which follows from the law of causality and not only from logical relations amongst concepts.

This is the basic difference between probability from the traditional mathematics (algebra, geometry, calculus) we inherited from the 19th century.

2. What is statistics?

Statistics is the scientific discipline which endeavors to acquire knowledge from experience with the objective to discover the laws of nature.

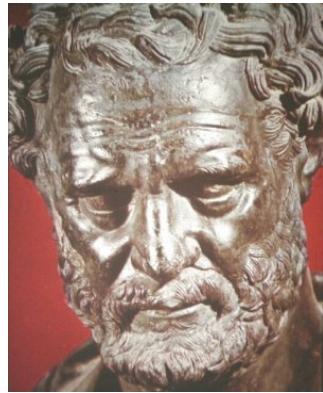
This obtains by means of observation, experimentation, and the application of the inductive method.

3. Relation between probability and statistics

Probability and statistics are closely related. Probability attempts predictions on the basis of certain stochastic models, and statistics checks the accuracy of predictions on the basis of data gathered by observation, or by way of experiments which itself designs.

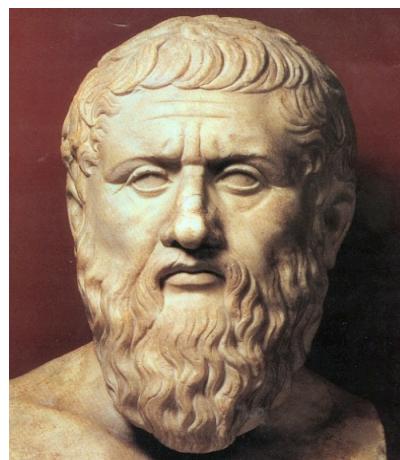
4. Classical antiquity-Hellenistic period

The Hellenic philosophy and science of the classical period were the pioneers of many later and contemporary developments in many areas. Geometry and astronomy are the obvious examples. The elementary use of some methods of calculus is another example. Also, the introduction of the concept of the atom by *Democritus* (460-370 B.C.) was a revolutionary idea with unpredictable and unanticipated consequences.



Democritus (460-370 B.C.)

It is somewhat peculiar, and certainly remarkable, that probability—and even less so statistics—were never subjects of study in the classical period. To be sure the concept of “probable” is found throughout the times from the period before *Socrates* (469-399 B.C.) and through the Hellenistic period (323- death of *Alexander the Great*-146 B.C., annexation of classical Greece by Rome). Philosophers and biographers such as *Democritus*, *Plato* (427-347 B.C.), *Aristotle* (384-322 B.C.), and *Plutarch* (46-120 A.D.) employed this term. In fact, *Plato* used the term “probability” in exactly the same way it is presently used in German (Wahrscheinlichkeit=similarity to something which is true).

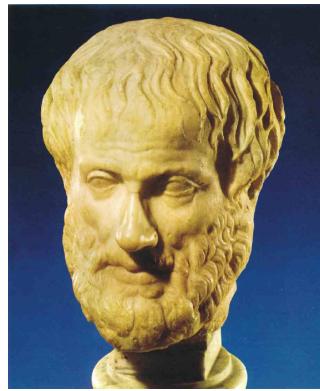


Plato (427-347 B.C.)

The classical Greeks were also aware of the fact that empirical truth is *not* based on the same solid foundation as mathematical truths, which are independent of time.

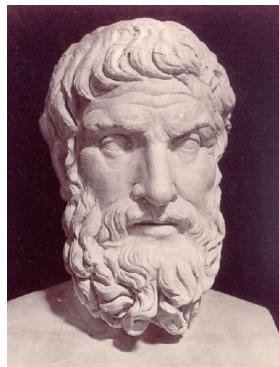
The *Stoic School* introduced and promoted the concept of *determinism* and *causal relationship* prevailing in the universe. The *Stoics* subscribed only to the concepts of the “necessary” and the “impossible”. According to them, the in-between concept of “possible” and that of “chance” are due simply to incomplete knowledge; all things happen according to preordained fate.

Aristotle did not fully subscribe to this position, and therefore in this sense, he was not absolutely an adherent of determinism.



Aristotle (384-322 B.C.)

On the other side, the *Epicurean School* espoused thoroughly *nondeterminism*, which was also introduced to the natural sciences. *Epicurus* (341-270 B.C.) himself subscribed to the atomic theory of *Democritus*, introduced new ideas into this theory, and helped advance it to the point that it became the pioneer of contemporary particle physics, quantum mechanics, and Brownian motion. It is not far fetched to also suggest that it contained the seeds of stochastic processes.



Epicurus (341-270 B.C.)

Although games of chance were known and practiced by the classical Greeks as well as Romans, Greeks did not show an interest in studying probability.

The main explanation for such an attitude may be found in the fact that the model of the classical Greek thought was the perfect functioning of the celestial bodies. Accordingly, the result of an action would be either “necessary” or “impossible”; there was no room left between them for the concept of the “probable”. It was impossible for them to accept that things could happen on earth in contradiction to the behavior of the heavenly bodies. So, there was no room for probability, which results from causality rather than being result of logical relations amongst ideas. To this, one must also add the almost entire lack of combinatorics, so closely related to elementary probability, as well as lack of algebraic symbols.

So much about probability.

Statistics would have no chance at all of being pursued to any degree, given that statistics is essentially an experimental discipline, and as is well-known, classical Greeks considered experimentation not a worthy pursue. Thus, the almost complete lack of experimentation did not allow them to capture the laws of chance which govern a *seemingly* chaotic series of events.

5. Probability in the 16th and the 17th centuries

The first probability calculations were made in the 16th century in Italy by the mathematician *Tartaglia* (1500-1557) and the mathematician and physicist *Gardano* (1501-1576). In France, the main contributors in probability at that time were *Pascal* (1623-1662) and *Fermat* (1601-1665).



Blaise Pascal (1623-1662)

Pascal was a mathematician of great influence, as well as a philosopher, and contributed to many areas of mathematics.



Pierre de Fermat (1602-1665)

Fermat practiced law as a profession and was an amateur mathematician. Together with *Descartes (1596-1650)* he laid the foundations of analytic geometry, contributed significantly to calculus, theory of numbers, and probability. He knew the Greek language well and, according to *Anders Hald*, the basis of *Fermat's* mathematics was the classical Greek works combined with the new algebraic methods of *Vieta (1540-1603)*. *Fermat* and *Pascal* are considered as the founders of probability.

However, the first book in probability was written in 1657 by *Huygens (1629-1695)* with *Pascal's* encouragement.



Christian Huygens (1629-1695)

Huygens studied law and mathematics and contributed to the development of modern calculus. He is best known, however, for his position on the wave nature of light.

Philosopher and mathematician *Leibnitz* (1646-1716) also contributed significantly to probability, although he is best known for his invention of calculus, simultaneously and independently of *Newton* (1643-1727). Also, he introduced the digital system which is the basis of the computer architecture.

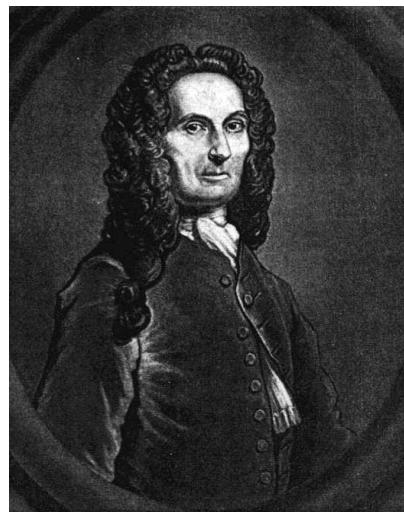
Nevertheless, the *mathematical* development of probability started in 1713 with the publication of *Jacob Bernoulli's* (1654-1705) work.



Jacob Bernoulli (1654-1705)

Among other things, he also introduced the concept of the Laws of Large Numbers.

De Moivre (1667-1754), who was born in France but lived in England, worked on various mathematical approximations to the binomial distribution, including that of normal approximation.



Abraham de Moivre (1667-1754)

This brings the closing of the 17th century from probability viewpoint.

6. Probability in the 18th century

The 18th century started with significant contributions to probability by great mathematicians of the European continent and a British mathematician.

In continental Europe, the most important mathematicians were *Laplace* (1749-1827) and *Gauss* (1777-1855).



Pierre Simon Laplace (1749-1827)

Laplace improved upon Bernoulli's and de Moivre's results, and had great influence in the applications of probability. Gauss is credited with the invention of the method of Least Squares, applied to astronomical data in 1809 within a probabilistic framework.



Karl Friedrich Gauss (1777-1855)

He is also credited with the discovery of the planet Poseidon whose existence was hypothesized, because of irregularities observed in the movements of the planet Uranus. Gauss studied the normal curve in conjunction with the method of Least Squares, but he did not invent it. Laplace was already aware of it, and it appears the credit of its discovery goes to de Moivre.



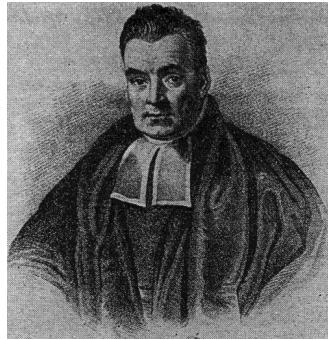
Siméon Denis Poisson (1781-1848)

Poisson (1781-1840) worked primarily in mathematical physics, and his most important contribution to probability was a paper published in 1837 in which he discussed what we now call *Poisson distribution*.

Lagrange (1736-1813) discovered the multivariate normal distribution as limit of the polynomial distribution; he also studied the distribution of the sample mean by means of moment generating functions.

Finally, the main contribution to probability by *Legendre* (1752-1833) was the discussion of the method of Least Squares with unsurpassed clarity and elegance.

On the other side of the English Channel, mathematician and presbyterian pastor *Bayes* (1702-1761) was one of the first mathematicians to occupy himself with statistical inference.



Thomas Bayes (1702-1761)

He formulated and proved with rigor and elegance his namesake theorem, which was promptly ignored by his contemporaries, it was published posthumously in 1764. It was two centuries after his death that it became widely known and formed the basis of the Bayesian inference.

7. Statistics in the 19th and 20th centuries

The 19th and the 20th centuries were the golden centuries for statistics, and the most significant advances took place in Great Britain.

Karl Pearson (1857-1936) was one of the pioneers.



Karl Pearson (1857-1936)

He studied mathematics in Cambridge University, and philosophy, physics, and law in Germany. In 1892, he published his “Grammar of Science” emphasizing the need for quantitative analysis in biology, medical science, and social sciences. He used statistics as a means of measuring the results of natural selection, and statistical methodology in studying evolutionary biology and eugenics. He played a leading role in the development of modern statistics, and emphasized correlation and curve fitting. Also, he discovered the Chi-square distribution. He established the *Biometrics Laboratory* with the help of *Yule* (1871-1951), and in 1901 published the first volume of *Biometrika* with the help of *Weldon* and *Sir Francis Galton* (1822-1911). *Weldon* (1860-1906) was an evolutionary biologist and founder of biometry.

Multifaceted scientific giant of about the same period was *Sir Ronald Fisher* (1890-1962), statistician, evolutionary biologist, eugenicist, and geneticist.



Sir Ronald Fisher (1890-1962)

According to *Anders Hald*, *Fisher* was a genius who almost by himself established the foundation of modern statistics, and according to the well-known biologist and author *Richard Dawkins* (former Professor at Oxford University, renown atheist and polemicist of religion), *Fisher* was the greatest of *Darwin's (1809-1882)* successors.

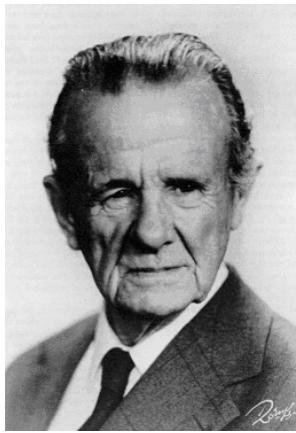
Fisher introduced the ANOVA in 1918, pioneered the introduction of experimental design, invented the method of likelihood, and introduced the concept of sufficiency, and of what we now call Fisher's information number. Also, he introduced the concept of randomized statistical tests, and essentially invented nonparametric statistics.

He visited the USA, and also India, where in Calcutta *P.C. Mahalanobis (1893-1972)*, a physicist and statistician, established the Indian Statistical Institute (ISI) with co-founders the mathematicians and statisticians *R.C. Bose (1901-1987)* and *S.N. Roy (1906-1964)*. ISI has been expanded widely and has become one of the top technological universities in India.



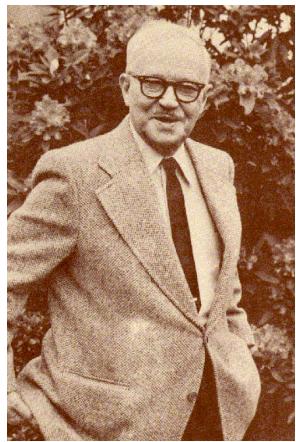
P.C. Mahalanobis (1893-1972)

Beyond *Pearson* and *Fisher* there were, of course, other worthy statisticians, such as *Yule*, *Galton*, and *Weldon* already mentioned, as well as *Gosset (1876-1926)*, *Edgeworth (1845-1926)*, *Egon Pearson (1895-1980)*, *Wishart (1898-1956)*, *Yates (1902-1994)*, and others. Here one must also mention the contributions to probability and statistics by the Swedish mathematician, actuary, and statistician *Harald Cramér (1893-1972)*.



Harald Cramér (1893-1972)

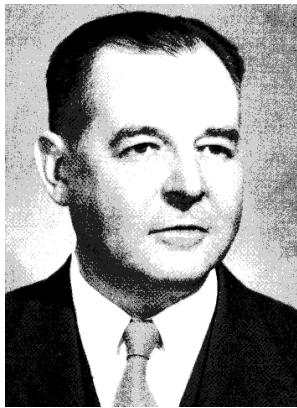
Another significant researcher of about the same period of time was the Polish mathematician *Jerzy Neyman (1894-1981)*.



Jerzy Neyman (1894-1981)

Neyman collaborated first with *Karl Pearson*, and subsequently with *Fisher* and *Egon Pearson*. He vigorously pursued the study of randomized experimental designs, in 1923, introduced the concept of confidence interval, in 1937, and together with *Egon Pearson* discovered their namesake Fundamental Lemma. In 1938, he went to UC-Berkeley where he stayed until his death in 1981.

In USA there was some statistical activity centered at the University of Iowa, where *H.L. Rietz (1875-1943)* taught mathematical statistics, and at Iowa State College of Agriculture (now Iowa State University, at Ames), where *G.W. Snedecor (1882-1974)* taught the first course in statistics in that university in 1915, and established the Iowa Statistical Laboratory in 1933, as well as the first department of statistics in USA. Later in 1944, there was established the Institute of Statistics at the University of North Carolina, Raleigh, consisting of the Department of Mathematical Statistics under *Hotelling (1895-1973)* and the Department of Applied Statistics under *Cochran (1909-1980)*. *Gertrude Cox (1900-1972)* was the first director of the institute, and the first to earn a Master's degree in Statistics from Iowa State College of Agriculture in 1931.



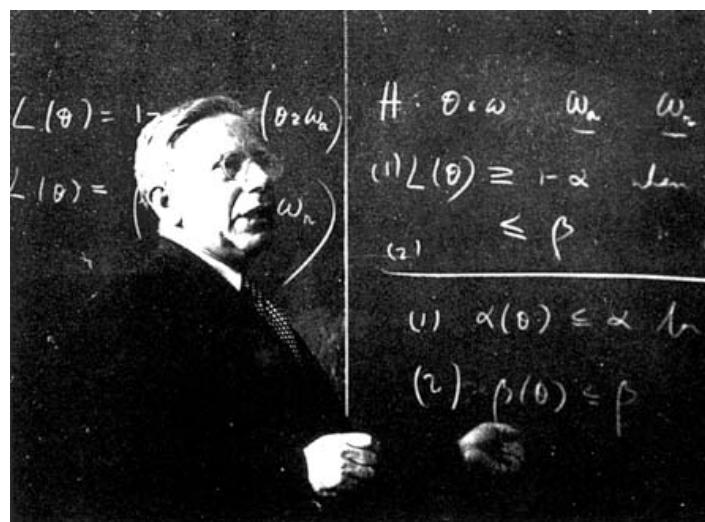
Harold Hotelling (1895-1973)

However, the golden era of statistics in USA commenced with the foundation of the Statistical Laboratory and the Department of Statistics at UC-Berkeley by *Jerzy Neyman*; recruiting for this purpose the best available talents in statistics and probability worldwide. The Statistical Laboratory was established in 1938, and the Department of Statistics in 1955 with essentially the first chair *David Blackwell (1919-2010)*.



David Blackwell (1919-2010).

Thus, *Neyman* created the top statistics department, which consistently ranks very high ever since. Another significant pioneer in the early development of statistics in USA was *Abraham Wald (1902-1950)* who came to the States around 1938.



Abraham Wald (1902-1950)

At first he carried out research for the Cowles Commission for Research in Economics, and later at Columbia University. *Wald* contributed decisively to decision theory, and founded sequential analysis.

In closing this section, it would be appropriate to mention George Boole (1815-1864), an English mathematician and philosopher, who contributed to probability-and indirectly to statistics-within a unified framework of mathematical logic and probability. He did this in his fundamental work “An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities” (1854). In this book, fully 6 out of 22 chapters are devoted to probability. His outlook on probability, as a component of a larger single structure, is summarized in the following quotation: “...no general method for the solution of questions in the theory of probabilities can be established which does not explicitly recognize...those universal laws of thought which are the basis of all reasoning...”

8. Probability in the 20th century

As for probability, it went through the stages as described above and was given its *final* mathematical formulation in 1933 by *Kolmogorov (1903-1987)* in his monograph *Foundation of the Theory of Probability*.

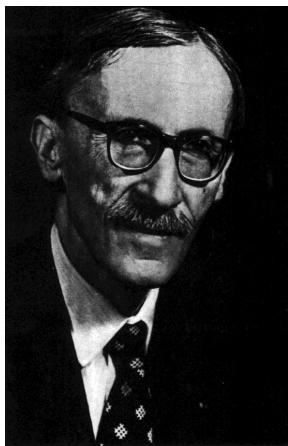


Andrei N. Kolmogorov (1903-1987)

By the *Consistency theorem*, he also founded the theory of *stochastic processes*. Stochastic processes introduced by *Markov (1856-1922)* and *Paul Lévy (1886-1971)* are among the most important processes.



Andrei A. Markov (1856-1922)



Paul Lévy (1886-1971)

However, the *first* rigorous foundation of probability took place in 1919 by *Carathéodory* in his paper [*C. Carathéodory, Ueber den Wienerkehrsatz von Poincaré, Sitz. Preuss. Akad. 11 Wiss. Phys. Math. 580-584 (1919)*].



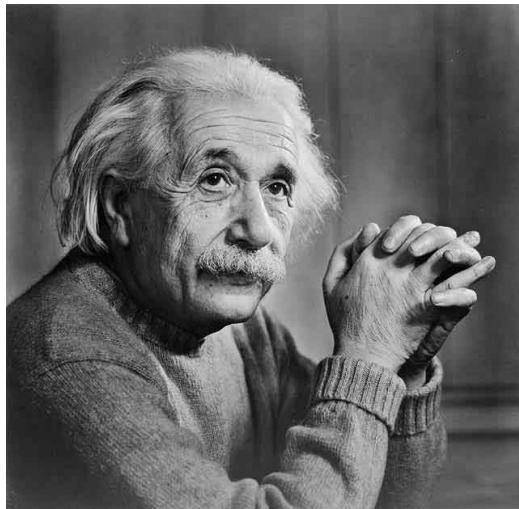
Constantine Carathéodory (1873-1950)

In the same paper, he also corrected some errors committed by the mathematician and theoretical physicist *Poincaré* (1854-1912) who applied probability incorrectly in the famous recurrence theorem in his paper [*H. Poincaré, Sur les tentatives d' explication mécanique des principes de la thermodynamique, Comptes rendus hebdomadaires de l' Académie des Sciences 108 (1889): 550-553. Sur le problème des trois corps et les équations de la dynamique, Acta Math. 13.1 (1890)*].



Henri Poincaré (1854-1912)

Carathéodory also contributed significantly in other areas, such as measure theory (outer measure, *Carathéodory* extension theorem, etc.), to the theory of functions, complex analysis, theory of variations, and thermodynamics. His pioneering paper on “Investigations on the Foundations of Thermodynamics” was enthusiastically received by *Max Planck (1858-1947)* and *Max Born (1882-1970)*. *Carathéodory* had kept long correspondence with *Albert Einstein (1879- 1955)*, and from information derived from *Einstein's* archives and made public by the State of Israel, it appears that *Carathéodory's* work helped *Einstein* in developing the theory of relativity.



Albert Einstein (1879-1955)

9. Statistics and probability in our times-Their influence to sciences

Since about the middle of the last century, statistics and probability have attained gigantic expansion and have penetrated all aspects of human activities.

Beyond population biology and genetics, which have already been mentioned, *modern biology* can hardly exist without statistical analysis. *DNA mapping*, e.g., and other biological procedures require perhaps as much biology as statistics.

Statistics plays a decisive role in extracting useful information from massive data without serious loss of information (*data mining*).

Cosmologists have arrived at the conclusion that about 25% of the mass in the universe consists of what is called *dark mass*, which helps keep the celestial bodies together, whereas about 75% of the energy in the universe is what is referred to as *dark energy*, and is considered the cause of the movement of the galaxies away from each other at tremendous speed. Statistics helps in studying the distribution of dark mass, and also in studying the properties of dark energy. Very relevant are two recent papers in the *Annals of Applied Statistics*, Vol. 3, No. 1, pages 6-37, by *Sarah Bridle et al.*, *Handbook for the greater challenge: An image analysis competition for cosmological lensing; and pages 144-178*, by *Christofer Genovese et al.*, *Inference for the dark energy equation of state using Type IA supernova data*.

In a recent copy of the news magazine “*Economist*” (October 31, 2009, page 84), it is observed that the backbone of the economic theory of the famous (infamous?) British economist *Keynes* is the indispensable uncertainty for the future.

It should come then as no surprise that economics proved to be fertile ground for the invasion of probability and statistics. *Mathematical economics* owns much to probability and stochastic analysis, and *econometrics* can be viewed essentially as application of statistics to economics entities.

It is then not an accident that, between 1969 and 2008, 20% of the Nobel Prizes awarded to economists were given to those working on probability/statistics, or topics closely related to them.

The applications of probability to *physics* have been profound. The physicist and mathematician *Clausius (1822-1888)* published in 1878 a paper on the kinetic theory of gases, using probability to arrive from a *seemingly* chaotic movement of an isolated small particle to an orderly behavior of a large set of such particles. This led physicists *Maxwell (1831-1879)* and *Boltzman (1844-1906)* to develop *statistical mechanics*.

Next, *Planck* adopted the idea that the levels of energy of a physical system may be discrete and described their distribution. Thus, he founded *quantum mechanics*.

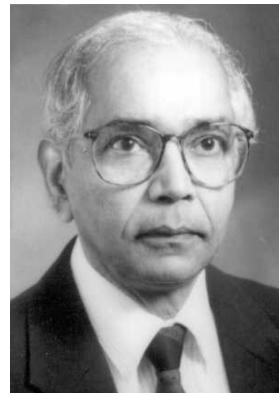
The theoretical physicist *Heisenberg* (1901-1976) also made significant contributions to quantum mechanics, but his fame derives from the *principle of uncertainty*, which he stated in 1927 and won the Nobel Prize in 1932. According to this principle, a moving particle is characterized by its position and the corresponding momentum (which is the product of mass and velocity). Measurements of these quantities, always contain an error, which is not due to the experimenter, but rather is an inherent feature of quantum mechanics. The *principle of uncertainty* states that the product of the two errors, in measuring the position and the momentum of a moving particle, cannot be smaller than a constant, the *quantum constant* ($\hbar/2\pi$); and this is essentially a probability statement.

It is said that *Einstein* was sufficiently upset by *Heisenberg's* findings as to state that it was inconceivable to him that God played with dice. But it seems that He does just that, in a certain sense!

It is an established fact that statistics attained its status as a scientific discipline in the Anglo-Saxon world. In the recent past and currently, probability and statistics are served by an army of talented probabilists and statisticians spread all over the world. It is also true that most of relevant activities do take place in the English speaking world (North American continent, U.K., Australia/New Zealand, and India). Just as an indication I mention the British-American mathematician and statistician *George E.P. Box* (1919-), the Indian-American statistician *C.R. Rao* (1920-), and the British statistician *Sir David Cox* (1924-).



George E.P. Box (1919-)



C.R. Rao (1920-)



Sir David Cox (1924-)

This section should not be concluded without referring briefly to John von Neumann and John Tukey.

John von Neumann (1903-1957) was a Hungarian-American mathematician who made major contributions to a vast range of fields. He is considered as one of the greatest mathematicians in modern history, and he was a legend in his own time. His contributions to probability and

statistics are by way of the book “Theory of Games and Economic Behavior” co-authored with Oskar Morgenstern in 1944, the invention of the statistical sampling technique Monte Carlo (“Monte Carlo Method” by Stan Ulam and John von Neumann (can you find the year?)), and the formulation of axioms for a probability-logic system (Memoirs of the American Mathematical Society, Volume 34 (1981)) which are motivated by quantum mechanics.

John W. Tukey (1915-2000) played a key role in the development and study of statistics in the mid 1900’s. He started as a chemist (Master’s degree in chemistry), earned a Ph.D. in mathematics on a topological topic, and turned to statistics by joining the war effort during World War II. His main contributions in statistics are in time series, exploratory data analysis, and multiple comparisons

10. Closing remarks

Probability and statistics are indispensable ingredients of human life and activities, due to the ever present uncertainty and complexity of nature. In such situations the principle of thermodynamics on equilibrium does not apply, but statistical regularity is present.

This is exactly what the physicist and chemist *Prigogine (1917-2003)* proved and was awarded the Nobel Prize in 1977.



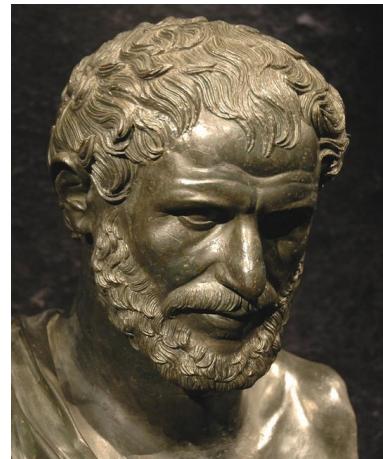
Ilya Prigogine (1917-2003)

Nowadays, the internet is perhaps one of the most complex systems, and as such it is fertile ground for the flourishing of probability and statistics.

Incidentally, what we now call World Wide Web (WWW) started in vastly different forms in the early 1960’s with pioneering work by *J.C.R. Licklider, Leonard Kleinrock, and Lawrence Roberts*, all of MIT, and within DARPA (Defense Advanced Research Project Agency). It went through many developmental stages and with many more participants, until in 1989 *Tim Berners-Lee* (an engineer and computer scientist) and others at CERN (the European Laboratory for Particle Physics) proposed the WWW protocol, with prodding and strong support by *Michael Dertouzos* of MIT. So, *Sir Timothy John Berners-Lee* came to be the founder of WWW as we know it today (rather than *Al Gore*-to use an inconvenient truth- despite of what you may have heard or read!)



Tim Berners-Lee (1955-)



Heraclitus (535-475 B.C.)

In March 2010, the establishment of a *Web Research Institute* was announced, to be operated jointly by Oxford and Southampton Universities. Let me close this presentation by quoting Heraclitus (535-475 B.C.) of the 4th century B.C. that “*everything flows and nothing abides; everything gives way and nothing stays fixed*”, or more briefly, “*Nothing is permanent, but change!*”; which in the vernacular becomes the familiar saying that *nothing in this world is certain except for death and taxes*. Everything else is subject to the laws of uncertainty. These laws, by the fact that they present statistical regularity, have been subjected to mathematical formulation in probabilistic language. Their discovery is the subject matter of statistics and is achieved by observation, experimentation, and the application of inductive reasoning.

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