STAT 131A MIDTERM II — SPRING QUARTER 2008 — Answers

1(a) Events A and B are independent if and only if $P(AB) = P(A)\,P(B)$.

1(b) (i) $P(X \le m)\,P(X \le n) = (m/6)\,(n/6) = mn/36$. (ii) $P(X \le m)^2 = (m/6)^2 = m^2/36$. (iii) Let E and F denote the events "$X \le 2$" and "$X \le 3$," respectively. Since $EF = E$ then

$$P(E \mid F) = P(EF)/P(F) = P(E)/P(F) = (2/6)/(3/6) = 2/3\,.$$

2(a) $E(X) = \sum_i x_i\,p(x_i)$, $E(X^2) = \sum_i x_i^2\,p(x_i)$, and $\text{var}(X) = E(X^2) - (EX)^2$ or

$$\text{var}(X) = \sum_i x_i^2\,p(x_i) - \Big\{\sum_i x_i\,p(x_i)\Big\}^2\,.$$

(Either of the latter two answers is acceptable.)

2(b) (i) $E(X) = -p+q+2\,(1-p-q) = 2-3p-q$. (ii) $E(X^2) = p+q+4\,(1-p-q) = 2-3\,(p+q)$. (iii) If $E(X) = 0$ and $\text{var}(X) = 1$ then $E(X^2) = 1$. Solving $E(X) = 0$ and $E(X^2) = 1$ for p and q gives $p = q = \frac{1}{2}$; or, simply say that the answer follows by symmetry and uniqueness.

3(a)(i) By expansion of the cubic (using the binomial theorem if necessary). (ii) Directly from (i) on noting that $E(X) = \mu$.

3(b) $E(X) = \displaystyle\int_{-\infty}^{\infty} x\,f(x)\,dx\,,\ E\big(X^2\big) = \int_{-\infty}^{\infty} x^2\,f(x)\,dx\,,\ E\big(X^3\big) = \int_{-\infty}^{\infty} x^3\,f(x)\,dx\,.$

3(c) By direct integration, $E(X) = \frac{1}{2}$, $E(X^2) = \frac{1}{3}$ and $E(X^3) = \frac{1}{4}$. Hence, $\text{var}(X) = \frac{1}{3} - (\frac{1}{2})^2 = 1/12$, $E(X-\mu)^3 = \frac{1}{4} - 3 \times \frac{1}{2} \times \frac{1}{3} + 2 \times (\frac{1}{2})^3 = 0$ (or simply, $E(X-\mu)^3 = 0$ by symmetry).