Solution for
Sample Midterm I

1. **WORK ALL PROBLEMS.** Please, write down formulas, give details and explanations and SHOW ALL YOUR WORK so that partial credits can be given.

2. You may use one page (two sides) of notes, and a calculator.

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1. The following data represents a random sample of eight inflation rates:

| Inflation Rate: | 1.6 | 2.2 | 2.3 | 4.5 | 6.1 | 5.6 | 5.1 | 4.6 |

(a) Calculate the mean and standard deviation of the rates.

\[
\bar{x} = \frac{1}{n} \sum x = \frac{1}{8} (32) = 4
\]

\[
S^2_x = \frac{1}{n-1} \sum (x-\bar{x})^2 = \frac{1}{7} (20.48) = 2.9543
\]

\[
S_x = \sqrt{2.9543} = 1.7188
\]

(b) Suppose 10% of the inflation is due to increase in gasoline prices. Find mean and variance of the inflation rates excluding the effect of the rise in gasoline prices.

\[
\gamma = .90 \bar{x}
\]

\[
\bar{\gamma} = .90 (\bar{x}) = .90 (4) = 3.6
\]

\[
S^2_\gamma = (.90)^2 S^2_x = (.90)^2 (2.9543) = 2.3930
\]

2. The following probability table is a breakdown on age and race/ethnicity of sport reporters:

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>&lt;35</th>
<th>35-44</th>
<th>45-54</th>
<th>55-64</th>
<th>&gt;65</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>.24</td>
<td>.20</td>
<td>.16</td>
<td>.12</td>
<td>.03</td>
</tr>
<tr>
<td>Non-White</td>
<td>.07</td>
<td>.08</td>
<td>.07</td>
<td>.02</td>
<td>.01</td>
</tr>
</tbody>
</table>

Define the following three events:

A: One randomly selected reporter is white.
B: One randomly selected reporter is 35-44 years old.
C: One randomly selected reporter is less than 35 years old.

Find \( P(A), P(B), P(C), P(A \text{ or } B), \) and \( P(A|C). \)

\[
P(A) = .24 + .20 + .16 + .12 + .03 = .75
\]

\[
P(B) = .20 + .08 = .28
\]

\[
P(C) = .24 + .07 = .31
\]

\[
P(A \cap B) = .20
\]

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \cap B) = .75 + .28 - .20 = .83
\]

\[
P(A \cap C) = .24
\]

\[
P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{.24}{.31} = .7742
\]
3. Even with strong advertising programs, new products are often unsuccessful. A company that produces a variety of household items found that only 20% of the new products it introduced over the last 10 years have become profitable. When two products were introduced during the same year, only 7% of the time did both products become profitable. Suppose the company plans to introduce two new products A and B next year. What is the probability that

(a) Product B will not become profitable?

\[ P(B^C) = 1 - P(B) \]
\[ = 1 - .20 = .80 \]

(b) At least one of the two products will become profitable?

\[ P(A \cup B) = P(A) + P(B) - P(AB) \]
\[ = .20 + .20 - .07 \]
\[ = .33 \]

(c) Neither of the two products will become profitable?

\[ P(A^C \cap B^C) = 1 - P(A \cup B) \]
\[ = 1 - .33 \]
\[ = .67 \]

(d) Product B will become profitable given that product A is profitable?

\[ P(B|A) = \frac{P(AB)}{P(A)} \]
\[ = \frac{.07}{.20} \]
\[ = .35 \]
4. A computer company is considering a plant expansion that will enable the company to begin production of a new computer product. The company’s president must determine whether to make the expansion a median- or large-scale project. Let X and Y denote the annual profit in $1,000. The following profit forecast has been developed for these two expansions:

<table>
<thead>
<tr>
<th>Demand</th>
<th>Median-Scale Expansion Profit</th>
<th>Large-Scale Expansion Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>x = 150 f(x) = .25</td>
<td>y = 110 f(y) = .20</td>
</tr>
<tr>
<td>Median</td>
<td>x = 250 f(x) = .45</td>
<td>y = 200 f(y) = .45</td>
</tr>
<tr>
<td>High</td>
<td>x = 300 f(x) = .30</td>
<td>y = 400 f(y) = .35</td>
</tr>
</tbody>
</table>

(a) Find the mean and standard deviation of the profits associated with each of the two expansion alternatives.

**Median-Scale**

\[
\mu_x = \sum x f(x) = 150(.25) + 250(.45) + 300(.30) = 240
\]

\[
\sigma_x^2 = \sum (x - \mu_x)^2 f(x) = (150-240)^2(.25) + (250-240)^2(.45) + (300-240)^2(.30) = 3150
\]

\[
\sigma_x = \sqrt{3150} = 56.1249
\]

**Large-Scale**

\[
\mu_y = \sum y f(y) = 110(.20) + 200(.45) + 400(.35) = 252
\]

\[
\sigma_y^2 = \sum (y - \mu_y)^2 f(y) = (110-252)^2(.20) + (200-252)^2(.45) + (400-252)^2(.35) = 12916
\]

\[
\sigma_y = \sqrt{12916} = 113.6486
\]

(b) Which decision is preferred for the objective of maximizing the expected profit? Which decision is preferred for the objective of minimizing the risk or uncertainty? Overall, which of these two options is preferred? Explain.

For max profit choose **Large-Scale** expansion.

For min risk choose **Median-Scale** expansion.

Overall, compare the Coefficient of Variations \(\frac{\sigma}{\mu}\).

**Median-Scale**

\[
\frac{\sigma_x}{\mu_x} = \frac{56.1249}{240} = .2339
\]

**Large-Scale**

\[
\frac{\sigma_y}{\mu_y} = \frac{113.6486}{252} = .4510
\]

Overall, choose **Median-Scale**.