(6-16) a) 

\[ P(x) \]

\[
\begin{align*}
\frac{1}{6} & \quad 1 \\
\frac{3}{6} & \quad 2 \\
\frac{2}{6} & \quad 3 \\
\frac{1}{6} & \quad 4
\end{align*}
\]

b) area under curve = \[ 1 \left( \frac{1}{6} \right) + 1 \left( \frac{4}{6} \right) + 1 \left( \frac{1}{6} \right) = 1 \]

c) \[ \Pr(0 < X < \frac{3}{4}) = 1 \frac{3}{4} \times \frac{1}{6} = \frac{3}{24} = 0.125 \]

d) \[ \Pr(X > 2.5) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} = 0.083 \]

e) \[ \Pr(0 < X < 1.5) = 1 \left( \frac{1}{6} \right) + 1 \left( \frac{4}{6} \right) + \frac{1}{6} = \frac{6}{12} = 0.5 \]

f) \[ \Pr(0.5 < X < 2.5) = 1 \left( \frac{1}{6} \right) + 1 \left( \frac{4}{6} \right) + \frac{1}{6} = \frac{1}{12} + \frac{4}{6} + \frac{1}{12} = \frac{5}{6} = 0.833 \]

(6-19) a) \[ \Pr(X > 14) = \Pr \left( \frac{X - \mu}{\sigma} > \frac{14 - 10}{\sqrt{24}} \right) = \Pr(Z > .82) = .2061 \]

b) \[ \Pr(8 < X < 20) = \Pr \left( \frac{8 - 10}{\sqrt{24}} < \frac{X - \mu}{\sigma} < \frac{20 - 10}{\sqrt{24}} \right) \]

= \[ \Pr(41 < Z < 2.04) \]

= \[ \left[ \Pr(0 < Z < .41) \right] \times \Pr(Z \geq 2.04) \]

= \[ 0.6591 \times 0.0207 = .006 \]

(6-5) c) \[ \Pr(|X - \mu| \geq 6) = \Pr \left( \frac{|X - \mu|}{\sigma} \geq \frac{6}{\sqrt{24}} \right) \]

= \[ \Pr(|Z| \geq 1.22) \]

= \[ 2 \Pr(Z \geq 1.22) = 2(0.1112) = .2224 \]
d) \( \Pr (X > X^*) = .10 \)
\[
\Pr \left( \frac{X - \mu}{\sigma} > \frac{X^* - 10}{\sqrt{24}} \right) = .10
\]
\[
\Pr (Z > \frac{X^* - 10}{\sqrt{24}}) = .10
\]
We know \( \Pr (Z > 1.28) \) is approximately .10.
Thus
\[
\frac{X^* - 10}{\sqrt{24}} = 1.28 \text{ and } X^* = (1.28)(\sqrt{24}) + 10 = 16.27
\]
e) \( Y \) is a linear function of the normal \( X \); thus \( Y \) is normal.

Note
\[
\begin{align*}
\mu_Y &= 4 + 6\mu_X = 4 + 6(10) = 64 \\
\sigma_Y^2 &= (6)^2\sigma_X^2 = (36)(24) = 864
\end{align*}
\]
Thus \( Y \sim N(64, 864) \)

So
\[
\Pr (Y > 16) = \Pr \left( \frac{Y - \mu_Y}{\sigma_Y} > \frac{16 - 64}{\sqrt{864}} \right)
= \Pr (Z > -1.63)
= 1 - \Pr (Z \leq -1.63) = 1 - .0516 = .9484
\]

\( \sqrt{6-21} \)
a) \( X \sim N \) with \( \mu = 15,000 \) and \( \sigma = 5000. \)
\[
\Pr (X < 8000) = \Pr \left( \frac{X - \mu}{\sigma} < \frac{8000 - 15,000}{5000} \right)
= \Pr (Z < -1.4) = \Pr (Z > 1.4) = .0808
\]

The poverty rate is then 8.08%.

b) Let \( X^* \) be that income such that \( \Pr (X > X^*) = .10 \)
\( X^* \) is then the 90th percentile. But if
\[
\Pr (X > X^*) = .10
\]
then
\[
\Pr \left( \frac{X - \mu}{\sigma} > \frac{X^* - 15,000}{5000} \right) = .10
\]
Since \( \Pr (Z > 1.28) \) is approximately .10, we know
\[
\frac{X^* - 15,000}{5000} = 1.28
\]
\[
X^* = (1.28)(5000) + 15,000 = $21,400
\]
(6-22) Since $r$ is normal, we know $l$ is normal
\[ \mu_l = 100,000,000 - 5,000,000 \quad \mu_r = 100,000,000 - 5,000,000 \]
\[ = 60,000,000 \]

\[ \sigma_l = 5,000,000 \quad \sigma_r = 5,000,000 \]

\[ \rightarrow \sigma_l = 5,000,000 \sigma_l = 5,000,000 \]

\[ \Rightarrow \sigma_l = 5,000,000 \sigma_l = 5,000,000 \]

\[ \Pr (l > 50,000,000) = \Pr \left( \frac{l - \mu_l}{\sigma_l} > \frac{50,000,000 - 60,000,000}{10,000,000} \right) \]
\[ = \Pr (Z > -1) \]
\[ = 1 - \Pr (Z > 1) \]
\[ = 1 - .1587 \]
\[ = .8413 \]

5. \( \lambda = 2, \) therefore, mean $= \frac{1}{2}=0.5, \) variance $= \frac{1}{4}=0.25 \]

6. \( \lambda = 3. \)

\[ \text{a. } P[x=2] = 0 \]
\[ \text{b. } P[x>2] = \exp((-3)*2)) = 0.002479 \]
\[ \text{c. } P[x<-4] = 1 - \exp((-3)*4) = 0.999993855 \]
\[ \text{d. } P[1<-x<-4] = P[x<-4] - P[x<1] = 0.999993855 - (1 - 0.049787) = 0.04978 \]

7. Mean $= 4,000, \) therefore, we have $\lambda = 1/4000 \)

\[ \text{a. } P[x > 1000] = 0.7788 \]
\[ \text{b. } P[x < 200] = 1 - 0.9512 = 0.0488 \]
\[ \text{c. } P[600 < x < 800] = 0.8607 - 0.8187 = 0.042 \]

8. Chi-square from table 3

\[ \text{a. 27.49} \]
\[ \text{b. 5.229} \]
\[ \text{c. 6.262} \]
\[ \text{d. 0} \]
(7-3) \( E(Q^d) = 50 \cdot .2E(P) = 50 \cdot .2(100) = 30 \)

(7-4) a) The \( p(X) \) is

<table>
<thead>
<tr>
<th>( X )</th>
<th>( p(X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{1}{6} )</td>
</tr>
</tbody>
</table>

\[
E(Y) = \sum_X (5 + 4X) p(X)
\]

\[
= (5 + 4(1))(\frac{1}{6}) + (5 + 4(2))(\frac{1}{6}) + (5 + 4(3))(\frac{1}{6}) + (5 + 4(4))(\frac{1}{6})
+ (5 + 4(5))(\frac{1}{6}) + (5 + 4(6))(\frac{1}{6})
\]

\[
= \frac{9}{6} + \frac{13}{6} + \frac{17}{6} + \frac{21}{6} + \frac{25}{6} + \frac{29}{6} = \frac{114}{6} = 19
\]

b) \( p(Y) \) is

<table>
<thead>
<tr>
<th>( Y )</th>
<th>( p(Y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>13</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>17</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>21</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>25</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>29</td>
<td>( \frac{1}{6} )</td>
</tr>
</tbody>
</table>

\[
E(Y) = \sum_Y Y p(Y) = 9(\frac{1}{6}) + 13(\frac{1}{6}) + 17(\frac{1}{6}) + 21(\frac{1}{6}) + 25(\frac{1}{6}) + 29(\frac{1}{6}) = 19
\]

c) \( E(Y) = 5 + 4E(X) = 5 + 4(3.5) = 19 \) since \( E(X) = 3.5 \)
\[ \sqrt{7.8} \quad \sigma^2 = E[ (X - E(X))^2 ] \]
\[ = E[ X^2 - 2E(X)X + [E(X)]^2 ] \]
\[ = E(X^2) - 2E(X)E(X) + [E(X)]^2 \]
\[ = E(X^2) - 2E(X)^2 + [E(X)]^2 \]
\[ = E(X^2) - [E(X)]^2 \]

Using the data from (7-5)
\[ \sigma^2 = E(X^2) - [E(X)]^2 \]
\[ = \sum_X X^2 p(X) - \left( \frac{\sum X}{2} \right)^2 \]
\[ = (1)^2 \left( \frac{1}{6} \right) + (2)^2 \left( \frac{1}{6} \right) + (3)^2 \left( \frac{1}{6} \right) + (4)^2 \left( \frac{1}{6} \right) + (5)^2 \left( \frac{1}{6} \right) + (6)^2 \left( \frac{1}{6} \right) - \left( \frac{7}{2} \right)^2 \]
\[ = 1 + 4 + 9 + 16 + 25 + 36 - \frac{49}{4} \]
\[ = \frac{91}{6} - \frac{49}{4} = \frac{35}{12} \]

\[ \sqrt{7.9} \quad E(Y) = 2E(X^2) + 4 \]

but, since
\[ \sigma^2 = E(X^2) - [E(X)]^2 \]

we know
\[ \sigma^2 + [E(X)]^2 = E(X^2) \]

and thus
\[ 9 + (5)^2 = E(X^2) \]
\[ E(X^2) = 34 \]

Thus, \( E(Y) = 2(34) + 4 = 72 \).