Problem 5.22  Let $E$ be the event that the person is a native English speaker, $S$ be the event that the person is a native Spanish speaker, $O$ be the event that the person is a native speaker of some other language, and $G$ be the event that the person is a high school graduate. Based on the given information, $\Pr(G|E) = 0.8$, $\Pr(G|S) = 0.9$, and $\Pr(G|O) = 0.6$.

(a)  We can write the event $G$ as the union of mutually exclusive events, as follows:

$$G = (G \cap E) \cup (G \cap S) \cup (G \cap O)$$

Then

$$\Pr(G) = \Pr(G \cap E) + \Pr(G \cap S) + \Pr(G \cap O)$$

$$= \Pr(G|E)\Pr(E) + \Pr(G|S)\Pr(S) + \Pr(G|O)\Pr(O)$$

$$= (0.8)(0.8) + (0.9)(0.15) + (0.6)(0.05)$$

$$= 0.64 + 0.135 + 0.03 = 0.805$$

So the probability that a randomly chosen person is a high school graduate is 0.805.

(b) Using the previous result, we have

$$\Pr(E|G) = \frac{\Pr(G \cap E)}{\Pr(G)} = \frac{\Pr(G|E)\Pr(E)}{\Pr(G)} = \frac{(0.8)(0.8)}{0.805} = 0.795$$

So the probability that a randomly chosen person who is a high school graduate is also a native English speaker is 0.805.

Problem 5.26  First make a table of the possible outcomes and the probability of each, using the fact that $\Pr(H) = 2/3$ and $\Pr(T) = 1/3$:

<table>
<thead>
<tr>
<th>outcome</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>HHH</td>
<td>$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$</td>
</tr>
<tr>
<td>HHT</td>
<td>$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{4}{27}$</td>
</tr>
<tr>
<td>HTH</td>
<td>$\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{4}{27}$</td>
</tr>
<tr>
<td>THH</td>
<td>$\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{27}$</td>
</tr>
<tr>
<td>HTT</td>
<td>$\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{2}{27}$</td>
</tr>
<tr>
<td>THT</td>
<td>$\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{27}$</td>
</tr>
<tr>
<td>TTH</td>
<td>$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{27}$</td>
</tr>
<tr>
<td>TTT</td>
<td>$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}$</td>
</tr>
</tbody>
</table>

(a) From the table, we determine that $\Pr(0 \text{ heads}) = 1/27$, $\Pr(1 \text{ head}) = 6/27$, $\Pr(2 \text{ heads}) = 12/27$, and $\Pr(3 \text{ heads}) = 8/27$. These results differ from those obtained with a normal coin since outcomes with many heads are more likely than outcomes with many tails.
(b) Note that $\Pr(A \cap B) = \Pr(A)$, since $A$ is a subset of $B$. Using the table and the rules, we have $\Pr(A) = \frac{8}{27}$, while

$$
\Pr(A\mid B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\Pr(A)}{\Pr(B)} = \frac{8/27}{18/27} = \frac{8}{18}
$$

Since $\Pr(A) \neq \Pr(A\mid B)$, events $A$ and $B$ are not independent.

(c) Following the same procedure, we have $\Pr(C) = \frac{2}{3}$ and

$$
\Pr(C\mid B) = \frac{\Pr(C \cap B)}{\Pr(B)} = \frac{\frac{8}{27} + \frac{4}{27}}{\frac{12}{27}} = \frac{\frac{12}{27}}{\frac{12}{27}} = \frac{2}{3}
$$

Since $\Pr(C) = \Pr(C\mid B)$, events $C$ and $B$ are independent.

**Problem 5.32** We use the given information to make a diagram, with the top circle representing basketball players ($B$) and the bottom circle representing tennis players ($T$). The left half represents males ($M$) and the right half represents females ($F$). The proportions for each possible combination of gender and sport are included.

![Diagram](image)

(a)

$$
\Pr(F\mid T) = \frac{\Pr(F \cap T)}{\Pr(T)} = \frac{(0.01)(0.45) + (0.05)(0.45)}{(0.01)(0.45) + (0.05)(0.45) + (0.02)(0.55) + (0.04)(0.55)}
$$

$$
= \frac{(0.06)(0.45)}{(0.06)(0.45) + (0.06)(0.55)} = 0.45
$$

So the probability of choosing a female, given that we have a tennis player, is 0.45.

(b) Note that $\Pr(\text{both teams} \cap T) = \Pr(\text{both teams})$, since $T$ is a subset of both teams. So

$$
\Pr(\text{both}\mid T) = \frac{\Pr(\text{both} \cap T)}{\Pr(T)} = \frac{\Pr(\text{both})}{\Pr(T)}
$$

$$
= \frac{(0.02)(0.55) + (0.01)(0.45)}{(0.02)(0.55) + (0.01)(0.45) + (0.04)(0.55) + (0.05)(0.45)}
$$

$$
= \frac{0.011 + 0.0045}{0.011 + 0.0045 + 0.022 + 0.0225} = \frac{0.0155}{0.06} = 0.2583
$$

So the probability of choosing a student who is on both teams, given that we have a tennis player, is 0.2583.
Pr(M|neither sport) = \frac{Pr(M \cap \text{neither sport})}{Pr(\text{neither sport})} = \frac{(0.90)(0.55)}{(0.90)(0.55) + (0.88)(0.45)} = \frac{0.495}{0.495 + 0.396} = \frac{0.495}{0.891} \approx 0.5556

So the probability of choosing a male, given that a student plays neither sport, is about 0.5556.

(d) From part (a), Pr(F|T) = 0.45, which is the same as Pr(F), so being on the tennis team is independent of being female. It must likewise be independent of being male. Therefore, being on the tennis team is independent of gender.

Problem 5.36 Let $S_1$ be the event that a business survives for one year, $S_2$ be the event that a business survives for two years, $F_1$ be the event that a business fails during the first year, and $F_2$ be the event that a business fails during the second year.

(a) $Pr(S_2 \cap S_1) = Pr(S_2|S_1)Pr(S_1) = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$

So the probability a new business will survive for two years is 4/9.

(b) $Pr(F_2 \cap S_1) = Pr(F_2|S_1)Pr(S_1) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{2}{9}$

So the probability a new business will fail in its second year is 2/9.

(c) Note that $Pr(\text{at least one survives}) = 1 - Pr(\text{all 5 fail within 2 years})$. For a single business, the probability it fails in either its first or second year is

$Pr(F_1 \cup (S_1 \cap F_2)) = Pr(F_1) + Pr(S_1 \cap F_2) = Pr(F_1) + Pr(F_2|S_1)Pr(S_1) = \frac{1}{3} + \frac{1}{3} = \frac{5}{9}$

So the probability that all five businesses fail within 2 years is $(5/9)^5 = 3125/59,049$. Therefore,

$Pr(\text{at least one survives}) = 1 - \frac{3125}{59,049} = \frac{55,924}{59,049} = 0.947$

This answer is based on the assumption that success or failure is independent among the businesses. This is probably a poor assumption since their success or failure is tied together by the state of the economy.
\( \mu = \sum_{x} x \cdot p(x) = 0(4) + 1(3) + 2(2) + 3(1) = 1.0 \)

\[ \sigma^2 = [\sum_{x} x^2 \cdot p(x)] - \mu^2 = 1^2(4) + 2^2(3) + 3^2(2) + 4^2(1) - (1.0)^2 = 1.0 \]

<table>
<thead>
<tr>
<th>( X )</th>
<th>( p(X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{1}{6} )</td>
</tr>
</tbody>
</table>

\[ \mu = 1(\frac{1}{6}) + 2(\frac{1}{6}) + 3(\frac{1}{6}) + 4(\frac{1}{6}) + 5(\frac{1}{6}) + 6(\frac{1}{6}) = \frac{31}{2} \]

\[ \sigma^2 = [\sum_{x} x^2 \cdot p(x)] - \mu^2 = 1^2(\frac{1}{6}) + 2^2(\frac{1}{6}) + 3^2(\frac{1}{6}) + 4^2(\frac{1}{6}) + 5^2(\frac{1}{6}) + 6^2(\frac{1}{6}) - \left(\frac{31}{2}\right)^2 \]

\[ = \frac{91}{6} - \frac{49}{4} = \frac{182 - 147}{12} = \frac{35}{12} \]

\( \sigma^2 = \sum_{x} (X - \mu)^2 \cdot p(X) \)

\[ = \sum_{x} [(x^2 - 2\mu x + \mu^2) \cdot p(x)] \]

\[ = \sum_{x} [x^2 \cdot p(x) - 2\mu \sum_{x} x \cdot p(x) + \mu^2 \sum_{x} p(x)] \]

\[ = \sum_{x} x^2 \cdot p(x) - 2\mu \sum_{x} x \cdot p(x) + \mu^2 \sum_{x} p(x) \]

\[ = \sum_{x} x^2 \cdot p(x) - 2\mu^2 + \mu^2 \quad \text{since} \quad \sum_{x} x \cdot p(x) = \mu \quad \text{and} \quad \sum_{x} p(x) = 1 \]

\( \sigma^2 = \sum_{x} x^2 \cdot p(x) - \mu^2 \)

\( n = 3, \quad \pi = \frac{1}{3} \)

\[ Pr(s = 1) = \frac{n!}{s!(n-s)!} \cdot \pi^s (1-\pi)^{n-s} = \frac{3!}{1!2!3^3} \left(1 - \frac{1}{3}\right)^{3} = \frac{12}{27} = 0.444 \]
\((6-10)\) \(\Pr(s \geq 1) = 1 - \Pr(s = 0) = 1 - \frac{4!}{0!4!}(0.56)^0(0.44)^4 = 1 - 0.0375 = 0.9625\)

\(\Pr(s \geq 2) = 1 - \Pr(s = 0) - \Pr(s = 1) = 1 - \frac{4!}{0!4!}(0.56)^0(0.44)^4 - \frac{4!}{1!3!}(0.56)^1(0.44)^3\)

\[= 1 - 0.0375 - 0.1908 = 0.7717\]

(6-11) \(n = 7, \pi = 0.2\)

\(\Pr(s \geq 4) = \Pr(s = 4) + \Pr(s = 5) + \Pr(s = 6) + \Pr(s = 7)\)

\[= 0.0287 + 0.0045 + 0.0004 + 0.0000\]

\[= 0.0334\]