Instructions: 1. **WORK ALL PROBLEMS.** Please, write down formulas, give details and explanations and **SHOW ALL YOUR WORK** so that partial credits can be given.

2. You may use three pages (two sides each) of notes, tables, and a calculator.

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1. (20 points) A manager believes that technology companies that have words such as cyber, link, and web in their name have lesser growth over the past year than technology companies that do not. To test this belief, a random sample of technology companies with and without these words in their titles were sampled and their percentage growth rates were recorded.

   With: 16.50 10.25 11.50 10.75 11.00 6.60 10.40
   Without: 18.75 11.00 9.75 10.50 10.00

   Do the data support the manager's belief?
   (a) State the null and alternative hypotheses.
   (b) Test at 0.05 level of significance. What assumptions are you making in this hypothesis testing?
   (c) Find the p-value.

2. (10 points) The amount of time required for a customer to check out at an express line at the Safeway supermarket is exponentially distributed with a mean time of 1.25 minutes.
   (a) Find the probability that a customer will require more than two minutes to check out.
   (b) What is the standard deviation of the time required for a customer to check out?

3. (25 points) Suppose that the tax, \( T_i \), expressed in thousands of dollars, paid by an individual \( i \) is a function of his or her income, \( X_i \), i.e.
   \[ T_i = .1(X_i - 50) \]
   \( i = 1, 2, \ldots, n \). The tax is expressed in thousand of dollars, so that there is a flat tax on all income over 50 thousand dollars. Suppose that individual incomes \( X_i \) are independently distributed with mean \( \mu = 30 \) thousand dollars and standard deviation \( \sigma = 10 \).

   (a) Find the probability that the incomes of three randomly chosen taxpayers all exceed 35 thousand dollars.
   (b) Determine the expected tax revenue received from a single taxpayer and its standard deviation.
   (c) If there are \( n = 10 \) individuals in a given area, determine the expected tax revenue received from the area and its standard deviation.
   (d) A sample of 9 returns from the entire population yielded a total tax of 25 thousand dollars. Determine a 95 percent confidence interval for the mean income in the area assuming incomes is normally distributed.

4. (20 points) Suppose \( X_1, X_2, X_3, X_4 \) and \( X_5 \) denote the daily sales of five randomly selected branches of a supermarket. To estimate average daily sales the following estimators were proposed.

   \[ \theta_1 = \frac{(X_1 + X_2 + X_3 - X_4 + X_5)}{4} \]
   \[ \theta_2 = \frac{(X_1 + X_2 - X_3 + X_4 + X_5)}{6} \]
   \[ \theta_3 = \frac{X_1 - X_2 + X_3 - X_4 + X_5}{6} \]

   (a) Which of the above estimators are unbiased? Justify your answer.
   (b) For the biased estimator(s) calculate the bias.
   (c) Suppose the mean and standard deviation of daily sales are $6,000 and $1,000, respectively. Using the mean square error criterion, order the three estimators in terms of their efficiency.
5. (25 points) A stockbroker for Price-Webber has a new client with $100,000 to invest in the market. Two stocks interest him but he wishes to invest in only one of them. The two stocks, denoted by Stock 1 and Stock 2, produced the summary statistics for percentage return over the past five years shown in the table below. We assume that percentage returns are independently and normally distributed with possibly unequal means and variances. Define the **sample risk ratio** as

\[ r = \frac{s_1^2}{s_2^2}, \]

the ratio of the sample variances. Let the **theoretical risk ratio** be defined by

\[ R = \frac{\sigma_1^2}{\sigma_2^2} \]

Use these definitions and the table given below to answer parts (a)-(c).

<table>
<thead>
<tr>
<th>Year</th>
<th>Stock 1 (%)</th>
<th>Stock 2 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>91</td>
</tr>
<tr>
<td>2</td>
<td>2148</td>
<td>1675</td>
</tr>
</tbody>
</table>

(a) If the theoretical risk ratio is \( R = 1 \), give the probability distribution of the sample risk ratio \( r \).

(b) If \( R = 1 \), find the number \( r^* \) such that \( P\{r \geq r^*\} = .05 \).

(c) Test the hypothesis that the theoretical risk ratio \( R = 1 \) for this data.

(d) Compare the average returns of the two groups by computing 90% confidence intervals for the mean returns \( \mu_1 \) and \( \mu_2 \).