Points

1. Meteorologists classify storms as either single or multiple peaks. The total number of lightning flashes was recorded for seven single-peak and five multiple-peak storms, resulting in the following data:

<table>
<thead>
<tr>
<th>Single-peak:</th>
<th>101</th>
<th>53</th>
<th>47</th>
<th>40</th>
<th>80</th>
<th>66</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple-peak:</td>
<td>227</td>
<td>201</td>
<td>245</td>
<td>239</td>
<td>208</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Does the data suggest that the true mean number of lightning flashes differ for the two types of storms? State the null and alternative hypotheses and test at 0.01 level.

\[
H_0: \mu_1 = \mu_2 \quad \text{vs} \quad H_A: \mu_1 \neq \mu_2 \quad \text{at} \quad 0.01 \text{ level}
\]

\[
\overline{Y}_1 = 66 \quad S_1 = 21.2446 \\
\overline{Y}_2 = 224 \quad S_2 = 19.1050 \\
\frac{S_1^2}{S_2^2} = 1.2309 < 2 \\
\text{Use Pooled Procedure}
\]

\[
S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}
\]

\[
= \frac{6 (21.2446)^2 + 4 (19.1050)^2}{7 + 5 - 2} = 416.8002
\]

\[
t = \frac{(\overline{Y}_1 - \overline{Y}_2) - 0}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}
\]

\[
= \frac{(66 - 224) - 0}{\sqrt{416.8002 \left( \frac{1}{7} + \frac{1}{5} \right)}} = -13.2171
\]

We can reject \( H_0 \) at 0.01 level. The mean number of lightning flashes are different for the two types of storms.
2. Two drugs A and B are compared for treatment of duodenal ulcer. In a random sample of 200 patients treated using drug A, 146 observed favorable results. In another random sample 200 patients treated by drug B, 165 observed favorable results.

(5) a. State the null and alternative hypotheses?

Let $p_1$ and $p_2$ represent proportion of favorable results using drug A and B, respectively. Then $H_0: p_1 = p_2$, $H_A: p_1 \neq p_2$

(15) b. Test your stated hypothesis at $\alpha = 0.05$ level and explain the results.

We can use a $\chi^2$ test.

<table>
<thead>
<tr>
<th>Drug</th>
<th>Favorable Result</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>146</td>
<td>54</td>
<td>200</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>165</td>
<td>35</td>
<td>200</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>311</td>
<td>89</td>
<td>400</td>
</tr>
</tbody>
</table>

\[
\chi^2 = \sum \frac{(O - E)^2}{E}
\]

\[
\chi^2 = \frac{(146-155.5)^2}{155.5} + \frac{(154-44.5)^2}{44.5} + \frac{(155-155.5)^2}{155.5} + \frac{(35-44.5)^2}{44.5}
\]

\[
\chi^2 = 5.2170
\]

d.f. = 1

We can reject $H_0$ at .05 level, the proportion of favorable results are different.

(5) c. What is the p-value? Explain the meaning of this p-value.

\[
.02 < p\text{-value} < .05
\]

The smallest level of significance for which we can reject $H_0$ is a value between .02 and .05. Since the p-value is less than .05, we can reject $H_0$ at .05 level.
3. The lead content of measurements taken at six Napa Valley wineries in two consecutive years are:

| First Year: | 0.08, 0.22, 0.34, 0.31, 0.39, 0.25 |
| Second Year: | 0.06, 0.25, 0.30, 0.35, 0.32, 0.20 |

Are these wineries improved in terms of reducing the lead levels.

(5) a. State the null and alternative hypotheses.

Let \( \mu_d \) denote the mean of differences between Year 1 and Year 2 measurements. Then, \( H_0: \mu_d = 0 \), \( H_a: \mu_d > 0 \)

(15) b. Assume the distribution of lead levels is not normal. Test your hypothesis at 0.05 level of significance.

We can use Wilcoxon Signed Rank Sum test

\[
d: \quad 0.02, -0.03, 0.04, -0.04, 0.07, 0.05
\]

\[
d\text{rl}: \quad 0.02, 0.03, 0.04, 0.04, 0.07, 0.05
\]

\[
\text{Rank}: \quad (1), (2), (3.5), (3.5), (6), (5)
\]

\[
W_- = 5.5, \quad \text{Note that } W_- + W_+ = \frac{n(n+1)}{2}
\]

\[
W_+ = 15.5 \quad 5.5 + 15.5 = 21 = \frac{6(7)}{2}
\]

\[
W_S = 15.5
\]

The critical value from Table 8 is 19

We cannot reject \( H_0 \) at 0.05 level.

\[
W_S = 15.5 \quad \text{Rejection: } 19 \quad \text{Reject } H_0
\]

(5) c. Find the p-value.

From Table 8, \( p\text{-value} > 0.10 \)
The American Dental Association is examining the effectiveness of two types of medication for reducing pain during dental work. A pain index is used to measure pain felt by the patients. The sample information is given below:

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Sample mean</th>
<th>Sample Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medication I</td>
<td>18</td>
<td>6.1</td>
</tr>
<tr>
<td>Medication II</td>
<td>20</td>
<td>8.7</td>
</tr>
</tbody>
</table>

Is there a difference between the mean pain indices of these two medications?

(5) a. Write down the null and alternative hypotheses.

\[ H_0 : \mu_1 = \mu_2 \quad \text{,} \quad H_A : \mu_1 \neq \mu_2 \]

(10) b. Test the hypothesis at 0.01 level of significance.

Assume the populations are normal and use the t procedure.

Since large \( \frac{s^2}{s_1} / \text{small} \frac{s^2}{s_2} = \frac{(2.9)^2}{(1.4)^2} = 4.2908 \), we use

\[
t = \frac{(\bar{Y}_1 - \bar{Y}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{6.1 - 8.7}{\sqrt{\frac{1.4^2}{18} + \frac{2.9^2}{20}}} = \frac{-2.6}{0.7276} = -3.5734
\]

\[
SE_1 = \frac{s_1^2}{n_1} = \frac{1.4^2}{18} = 0.1089 \quad SE_2 = \frac{s_2^2}{n_2} = \frac{2.9^2}{20} = 0.4205
\]

\[
df = \frac{(SE_1^2 + SE_2^2) \left( \frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} \right)}{(SE_1^2 / (n_1 - 1)) + (SE_2^2 / (n_2 - 1))} = 28.02 
\]

We can reject \( H_0 \) at .01 level.

(5) c. Find the p-value.

\[ 2(.0005) < p-value < 2(.005) \]

\[ .001 < p-value < .01 \]

(10) d. Construct a 99% confidence interval for the difference between the two means and explain its meaning. Does this confidence interval support the null hypothesis? Explain your answer.

The 100(1-\( \alpha \))% C.I. for \( (\mu_1 - \mu_2) \) is \( (\bar{Y}_1 - \bar{Y}_2) \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \)

\[ d = .01, \quad \alpha/2 = .005, \quad df = 28, \quad t .005 = 2.763. \quad \text{Then,} \]

the 99% C.I. is \( (6.1 - 8.7) \pm 2.763 (0.7276) \) or

\[ -2.67 \pm 2.0104 \quad \text{or} \quad [-4.6104, -0.5896] \]

Since 0 is not inside the 99% confidence interval, we can reject \( H_0 \) at .01 level.