Statistics 100

Sample
Midterm I

Instructions: 1. WORK ALL PROBLEMS. Please, give details and explanations and SHOW ALL YOUR WORK so that partial credits can be given.

2. You may use one page of notes and a calculator but no other reference materials.

Points

1. A survey was conducted to investigate the long-term prognosis of children who have suffered an acute episode of bacterial meningitis. The following data are the times (in minutes) to the onset of seizure for 15 children.

   34, 2, 12, 39, 16, 7, 9, 63, 23, 15, 0, 19, 6, 12, 43

   (10) (a) Use the stem-and-leaf plot to compute the sample quartiles.

   0 | 0 2 6 7 9
   1 | 2 2 5 6 9
   2 | 3
   3 | 4 9
   4 | 3
   5 |
   6 | 3

   \[
   Q_1 = \frac{7+9}{2} = 8
   \]

   \[
   Q_2 = \text{Median} = 15
   \]

   \[
   Q_3 = \frac{23+34}{2} = 28.5
   \]

   (10) (b) Compute sample mean, variance and standard deviation.

   \[
   \bar{y} = \frac{\sum y}{n} = \frac{300}{15} = 20
   \]

   \[
   s^2 = \frac{\sum (y-\bar{y})^2}{n-1} = \frac{4324}{15-1} = 308.8571
   \]

   \[
   s = \sqrt{308.8571} = 17.5743
   \]
2. If one crosses garden peas having gene pair (red, white) with other peas also have gene pair (red, white), one-fourth of the progeny are expected to have white flowers. In 64 plants from such a cross, what is the probability that:

(a) Fewer than 16 will have white flowers?

Let \( y \) denote the number of plants with white flowers.

Then \( y \) is distributed as binomial with \( n = 64 \), \( p = .25 \).

Since \( np = 16 > 5 \) and \( n(1-p) = 48 > 5 \), then \( y \) is approximately normal with mean \( \mu = np = 64(.25) = 16 \)
and standard deviation \( \sigma = \sqrt{np(1-p)} = \sqrt{64(.25)(.75)} = 3.4641 \).

\[
P[ y < 16 ] \approx P[ y \leq 15.5 ]
\]
\[
= P[ Z \leq \frac{15.5 - 16}{3.4641} ]
\]
\[
= P[ Z \leq -1.443 ]
\]
\[
= .4443
\]

(b) More than 10, but less than or equal to 20 will have white flowers?

\[
P[ 10 < y \leq 20 ] \approx P[ 10.5 \leq y \leq 20.5 ]
\]
\[
= P\left[ \frac{10.5 - 16}{3.4641} \leq Z \leq \frac{20.5 - 16}{3.4641} \right]
\]
\[
= P \left[ -1.5877 \leq Z \leq 1.2990 \right]
\]
\[
\approx P \left[ -1.59 \leq Z \leq 1.30 \right]
\]
\[
= .9032 - .0559
\]
\[
= .8473
\]
3. Suppose that two out of five women who become pregnant with the aid of fertility drugs experience multiple births (twins, triplets, and soon). In a randomly selected group of 5 women, who became pregnant with the assistance of fertility drug, find the probability that:

(7) (a) None will have multiple births.

We have a binomial random variable with \( n = 5, p = .4 \).

\[
P(\gamma = 0) = \binom{5}{0} (0.4)^0 (0.6)^5
\]
\[
= \frac{5!}{0! \cdot 5!} (0.6)^5
\]
\[
= 0.0778
\]

(6) (b) Not more than one will have multiple births.

\[
P(\gamma \leq 1) = P(\gamma = 0) + P(\gamma = 1)
\]
\[
= \binom{5}{0} (0.4)^0 (0.6)^5 + \binom{5}{1} (0.4)^1 (0.6)^4
\]
\[
= 0.0778 + \frac{5!}{1! \cdot 4!} (0.4) (0.6)^4
\]
\[
= 0.0778 + 0.2592
\]
\[
= 0.3370
\]

(7) (c) Less than 5 will have multiple births.

\[
P(\gamma < 5) = 1 - P(\gamma = 5)
\]
\[
= 1 - \binom{5}{5} (0.4)^5 (0.6)^0
\]
\[
= 1 - 0.0102
\]
\[
= 0.9898
\]
4. The length of time it takes variety of seeds of a plant to germinate is normally distributed with mean of 15 days and variance of 16 days.

(6) (a) What proportion of the seeds should germinate within 19 days.

\[ P[\gamma \leq 19] = P[Z \leq \frac{19-15}{4}] = P[Z \leq 1] = 0.8413 \]

84.13% of the seeds should germinate within 19 days.

(6) (b) What proportion between 10 to 19 days?

\[ P[10 \leq \gamma \leq 19] = P\left[\frac{10-15}{4} \leq Z < \frac{19-15}{4}\right] = P[-1.25 \leq Z \leq 1] = 0.8413 - 0.1056 = 0.7357 \]

73.57%.

(7) (c) By what day should 95% of the seeds have germinated?

\[ Z = \frac{\gamma - \mu}{\sigma} = \frac{\gamma - 15}{4} = 1.645 \]

\[ \gamma = 1.645 \times 4 + 15 \]

\[ \gamma = 21.58 \text{ days}. \]