Comparison of Two Independent Populations (continued)

III. Inferences When $\sigma_1$ and $\sigma_2$ are Unknown

A Practical Problem: Suppose we are interested in comparison average systolic blood pressures (SBP) between males and females 45-50 years old. We collect the following data:

Male: 130 134 137 135 140 134
Female: 127 138 125 134

Question: Is SBP on average higher for males compared to females?

a. If the two populations are normal with unknown variances, then

b. A $100(1 - \alpha)\%$ confidence interval for $(\mu_1 - \mu_2)$ when the population standard deviations are unknown is:

Example: Systolic Blood Pressure
c. The *pooled two-sample procedure*

If \( \sigma_1 \) and \( \sigma_2 \) are unknown but \( \sigma_1 = \sigma_2 \), then a *pooled estimate* of the common variance is:

Using this estimate, a \( 100(1 - \alpha) \)% confidence interval for \( (\mu_1 - \mu_2) \) is

**Example:** Orange Juice Study
We would like to compare the taste of a “new” product to an “old” product based on samples of size 15 and 14. Suppose the mean of the first sample is 9.5 with a standard deviation of 1.3, and the mean of the second sample is 7.1 with a standard deviation 1.5.
IV. Hypothesis Testing

a. Null and Alternative Hypotheses

*Null Hypothesis:* A conjecture about a parameter of a population or parameters of two population.

*Example:* Systolic Blood Pressure

*Alternative Hypothesis:* Another conjecture about the same parameter(s).

*Example:* Systolic Blood Pressure

b. Test Statistics

The decision to reject or not reject the null hypothesis is based on a *statistic* computed from the sample. This is called the *test statistic*.

c. Use of Data and Possible Errors

The hypothesis will be tested based on information from *data*.

Possible Errors:

<table>
<thead>
<tr>
<th>Actual</th>
<th>H₀ is true</th>
<th>H₀ is false</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject H₀</td>
<td></td>
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<tr>
<td>Decision</td>
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<tr>
<td>Based on</td>
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<tr>
<td>Data</td>
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<tr>
<td>Do not</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reject H₀</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

α :
1 - β :
d. The t-test

To test \( H_0 : \mu_1 = \mu_2 \) compute \( t \)

and reject \( H_0 \) in favor of \( H_A \) of

\( H_A : \mu_1 < \mu_2 \) if \( t < -t_\alpha \)

\( H_A : \mu_1 > \mu_2 \) if \( t > t_\alpha \)

\( H_A : \mu_1 \neq \mu_2 \) if \( t < -t_{\alpha/2} \) or \( t > t_{\alpha/2} \)

**Example:** Systolic Blood Pressure
d. The *pooled two-sample procedure*

If $\sigma_1$ and $\sigma_2$ are unknown but $\sigma_1 = \sigma_2$, then use the *pooled Procedure*.

**Example:** The Orange Juice Study

*p-value:*