Probability

I. Introduction

a. What is the use of probability?

- Probability allows us to handle variations in experimental outcomes mathematically. It helps us to deal with uncertainties.

  Examples: Risk of breast cancer.  
  Chance of being hit by a deadly virus.  
  Tuberculin skin tests results.

- Statistical inference is based on probability.

b. Experiment: Process of observing a random phenomenon.

  Examples: Coin tossing.  
  Selecting an item at random to test for quality.  
  Select an individual at random and record the blood type.

c. Sample Space: The set of all possible distinct outcomes.

  Examples: Toss a coin once  \( S = \{H, T\} \).  
  Roll a die once  \( S = \{1, 2, 3, 4, 5, 6\} \).  
  Select an item at random  \( S = \{\text{good, defective}\} \).  
  Select an individual at random and record the blood types  \( S = \{\text{A, AB, B, O}\} \).

d. Event: The set of all outcomes having a certain feature.

  Examples: Toss a coin once  \( E = \{H\} \).  
  Roll a die once  \( E = \{2, 4, 6\} \).  
  Select an item at random  \( E = \{\text{defective}\} \).  
  Blood types  \( E = \{\text{A, AB}\} \).
II. Assigning Probabilities to Events

a. Classical Approach: Suppose the sample space consists of \( n \) distinct outcomes. If the outcomes are equally likely, then

\[
P(E) = \frac{\text{Number of outcomes in } E}{n}
\]

Examples:

b. Relative Frequency Approach: Probability of an event is a number representing the portion of times the event is expected to occur when the experiment is repeated many times.

Examples:

c. Subjective Approach: Subjective probability reflects the degree to which we believe the event will occur.

Examples:
d. Probability Rules


e. Probability Trees
   Example:

f. Probability of Combination of Events
III. Conditional Probability and Independence

a. Let A and B be two events such that \( P(B) \neq 0 \). The \textit{conditional probability} of A given B is defined as:

\[
P(A|B) = \frac{P(A \text{ and } B)}{P(B)}
\]

\textbf{Example:} Side-effects

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<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>None</td>
<td>60</td>
<td>80</td>
<td>140</td>
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<tr>
<td>Mild</td>
<td>20</td>
<td>10</td>
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<tr>
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<td>5</td>
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</tr>
<tr>
<td>Severe</td>
<td>5</td>
<td>5</td>
<td>10</td>
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Total   100   100    200

A = No side-effect.
B = Selected individual is male.

b. Multiplication Rule:

Use the definition of conditional probability:

\[
P(A \text{ and } B) = P(A) \ P(B|A) = P(B) \ P(A|B)
\]

\textbf{Example:} What is the probability of an oil tanker to involve in an accident and produce a major oil spill?
c. Independence:

Events A and B are independent if

\[ P(A|B) = P(A) \quad \text{or} \quad P(B|A) = P(B). \]

This leads to

\[ P(A \text{ and } B) = P(A) \cdot P(B). \]

**Example:** Side-effects

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A = No side-effect.
B = Selected individual is male.
C = Severe side-effects.