Statistics 106: Analysis of Variance

Second Examination Solutions

Take Home November 19 - November 24

Fall Quarter, 2002

1. (10 pts) *Write out the two linear models used to test the significance of gender (Factor A) and estimate the error sum of squares for a model that does not include gender (it is not SSA).*

   The three-factor model is that the mean value of the data is of the form

   $$\mu_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk}$$

   where the data are normal, independent, with equal variances. The main effects and interactions above are constrained to sum to zero over any single subscript.

   H_0: All α_i in the three-factor model are zero

   H_a: Some α_i in the three-factor model are non-zero.

   The sum of squares due to A is the difference between the error sum of squares under H_0 above and the error sum of squares under under H_a, i.e. $SSA = SSE(H_0) - SSE$, since SSE in the ANOVA is the error sum of squares under the full model H_a. Therefore $SSE(H_0) = SSA + SSE = 540,361 + 41,186 = 581,547$

2. (25 pts) *Construct the ANOVA, testing all factor combinations at an overall level of* $\alpha = .05$ *using the Kimball Inequality. Interpret your results in practical terms.*

   To test all factors, we need 7 F-tests, so that the F-levels for the separate tests will be approximately given by using the α solving $1 - (1 - \alpha)^7 = .05$ or $1 - \alpha = .95^{(1/7)} = .9927$. We need $F(.993, 1, 48)$ and $F(.993, 2, 48)$ Close values in the table are $F(.995, 1, 60) = 8.49$ and $F(.995, 2, 60) = 5.80$ Interpolation between 30 and 60 is acceptable, as is using the value of 30 for error degrees of freedom. All F^* values for main effects in the table below exceed the critical values, whereas no interaction terms are significant. Gender and experience seem to be more significant contributors than the sequencing method.

ANOVA

Source	df	SS	MS	F^*
Gender	1	540,361	540,361	630
Sequence	2	49,320	24,660	29
Experience	1	382,402	382,402	446
GS	2	543	271	.21
GE	1	91	91	.11
SE	2	911	456	.53
GSE	2	19	10	.01
Error	48	41,186	858	
Total	59	1,014,832		

3. (25 pts) *Set up comparisons for the main effects of gender, assembly sequence, and months of experience on assembly time using Bonferroni intervals with an overall level of* $\alpha = .05$.

   Since the F-tests only give significance and do not give information about the differences in the assembly time under the various factors, we use multiple comparisons to zero in on the differences due to gender, experience and sequence. The mean differences of interest are

   Gender: $\bar{Y}_{1\ldots} - \bar{Y}_{2\ldots} = 1156 - 966 = 190$

   Sequence (1 vs 2): $\bar{Y}_{.1..} - \bar{Y}_{.2..} = 1044 - 1101 = -57$

   Sequence (1 vs 3): $\bar{Y}_{.1..} - \bar{Y}_{.3..} = 1044 - 1037 = 7$

   Sequence (2 vs 3): $\bar{Y}_{.2..} - \bar{Y}_{.3..} = 1101 - 1037 = 64$

   Experience: $\bar{Y}_{..1.} - \bar{Y}_{..2.} = 1141 - 981 = 160$

   So there are $g = 5$ comparisons, leading to $t(1 - .05/10, 48) = t(.995, 48) \approx 2.68$ Note that 2.66 and 2.70 are OK here as well. Note that $i = 1, 2, j = 1, 2, 3, k = 1, 2$, with $n = 5$ observations in each cell mean. Hence, gender and experience means each have 3(2)(5)=30 observations and the sequence means each have 2(2)(5)=20 observations. Hence, the two standard deviations to use are $s_1(\hat{D}) = \sqrt{858(2)/30} = 7.55$ and $s_2(\hat{D}) = \sqrt{858(2)/20} = 9.26$, and we have $ts_1 \approx 20$ for sequence comparisons and $ts_2 \approx 25$ for gender and experience. Looking at the mean differences above shows that females take an average of 190 minutes less than males, with a simultaneous 95% confidence interval for superiority of 165 to 215 minutes. The lowest average sequence assembly times are for methods 1 and 3 and they are both significantly better than method 2 although not significantly different from each other. Experience is important, with a difference of 160 minutes (135-185 95% confidence interval).

4. (15 pts)*Plot the mean profiles for males and females separately over both sequence and experience. Interpret your results in light of what was obtained in questions 2. and 3.*

   First Plot

   Males over Sequence: 1135 1198 1135

   Females over Sequence: 954 1005 940

   Second Plot

   Males over Experience: 1237 1075

   Females over Experience: 1044 888

   The plots are parallel indicating that neither gender and sequence or gender and experience interact, i.e., with added experience or changed methodology, neither the mens' or womens' assembly time increases more than would be expected from the simple additive model. This confirms the lack of significance of the GS and GE terms in the ANOVA.

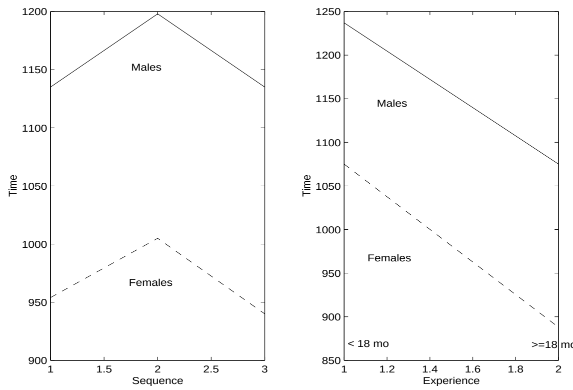

Figure 1: Male and female profiles as a function of sequence and experience

5. (25 pts) *Suppose that there are an equal number of males and females in the work force and that only 25% of the work force have 18 or more months of experience. Find the best assembly sequence to use and estimate the overall mean assembly time for this sequence. Give a 95% confidence interval for this sequence using Bonferroni's method and taking into account that you are using more than one possible linear combination.*

   We want to find the best assembly sequence and develop a 95% confidence interval for the mean assembly time that would be expected for the composition of our work-force if we decide to use only one sequence. The means are

   Sequence 1: .75(.50)1219+.75(.50)1036+.25(.50)(1051)+.25(.50)871=1086

   Sequence 2: .75(.50)1274+.75(.50)1077+.25(.50)(1122)+.25(.50)932=1138

   Sequence 3: .75(.50)1218+.75(.50)1020+.25(.50)(1051)+.25(.50)860=1078.

   The standard deviations of the estimated linear functions are

$$s(\hat{L}) = \sqrt{858\left(\frac{.375^2}{5} + \frac{.375^2}{5} + \frac{.125^2}{5} + \frac{.125^2}{5}\right)} = 7.32$$

   with $t(1-.05/6, 48) = t(.992, 48) \approx 2.52$, with 2.52 and 2.54 also acceptable. Now, $ts = 2.52(7.32) \approx 18$. Again, sequences 1 and 3 give the best average work-force assembly time with confidence intervals 1086 ± 18 minutes and 1078 ± 18 minutes respectively. Both these are significantly better than 2, which would have a mean work-force assembly time of 1138 ± 18 minutes.